Affecting Distributions of Opinions with Evidence

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This Paper

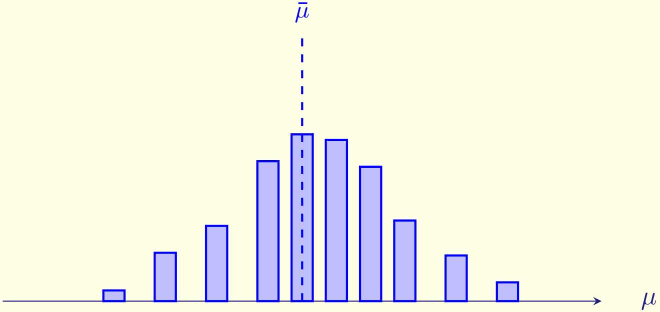
Basic consideration 1: communicators care about features of distributions of opinions: Adams et al. (2024); Sommer-Topcu (2009); Lawrence et al. (2011); Pereira (2009); Monroe (1998); Walgrave et al. (2022); Durovic and Schanatterer (2025)

Beliefs elicited? theory for lab, **Brier 1950's elicitation**; in practice, opinion surveys, popular consultation, feedback from plebiscites

Basic consideration 2: partial provability $\rightarrow$ relaxes extreme skepticism (e.g., Milgrom and Roberts, 1986)

From Frequency to Theoretical Distributions

$\mu = \mathbb{P}(\omega = 1)$



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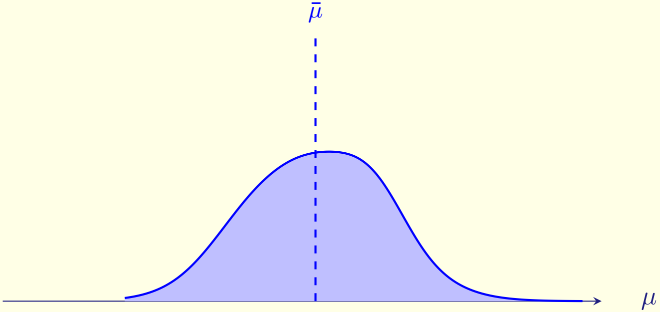


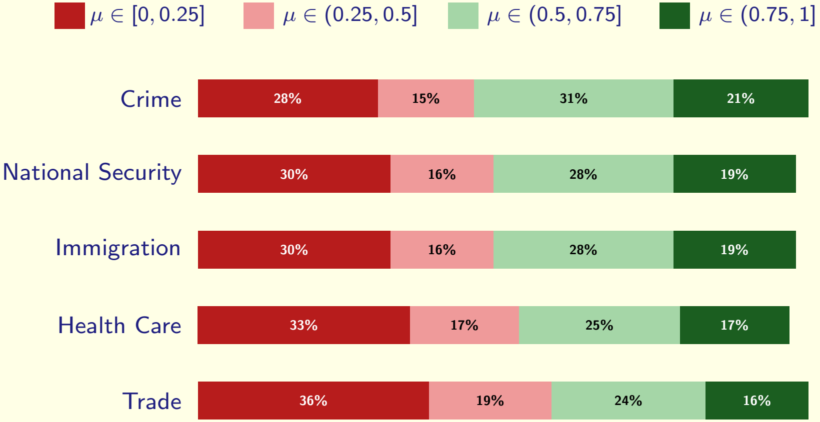
Motivation

How do evidence acquisition and disclosure affect distributions of opinions on unknown issues?

- Knowledge about communicator's efforts $\rightarrow$ audience's skepticism?
- Value of evid. acquisition and disclosure to communicators?
- Welfare implications for audiences?
- Role of evid. provision technology?, ... of initial distributions of opinions?

Motivation

Opinions $\mu = \mathbb{P}(\omega = 1)$ that Trump's policy-making is **good** $(\omega = 1)$
(Economist/YouGov Poll, Apr 25–May 2, 2025)



Motivation

Opinions on **good** ($\omega = 1$) effects of tariffs:

The New York Times **2024 ELECTIONS** | Cross-Tabs: September 2024 Times/Siena Polls in Michigan, Ohio and Wisconsin Share full article

Expanding tariffs on products made outside the United States.	MI, OH AND WI LIKELY ELECTORATE	MI	OH	WI	MEN	WOMEN	18-29	30-44	45-64	65+	WHITE	BLACK	OTH.	B.A.+	NO B.A.	WHITE, COLLEGE	WHITE, NO COLLEGE	NON-WHITE, COLLEGE	NON-WHITE, NO COLLEGE	CITY
Strongly support	29%	33%	30%	25%	37%	22%	19%	24%	33%	32%	31%	15%	26%	20%	34%	20%	36%	20%	22%	20%
Somewhat support	25%	24%	27%	24%	22%	28%	36%	28%	23%	21%	26%	12%	24%	28%	24%	29%	25%	22%	17%	20%
Somewhat oppose	17%	16%	15%	20%	16%	18%	24%	20%	17%	13%	16%	28%	16%	22%	15%	22%	13%	21%	22%	21%
Strongly oppose	21%	20%	20%	22%	20%	22%	11%	19%	21%	27%	20%	32%	24%	23%	20%	22%	19%	28%	28%	25%
[VOL] Don't know/Refused	8%	7%	8%	9%	5%	10%	10%	10%	5%	8%	7%	12%	10%	7%	7%	7%	7%	10%	12%	14%
NET Support	54%	57%	57%	49%	59%	50%	55%	51%	56%	53%	57%	28%	51%	48%	58%	50%	61%	41%	39%	41%
NET Oppose	38%	36%	36%	42%	36%	40%	35%	39%	38%	40%	36%	60%	39%	44%	35%	44%	32%	49%	50%	45%
Number of respondents	1,690	557	567	566	800	865	261	338	589	457	1,370	133	147	731	950	592	775	122	158	302
Percentage of total electorate	100%	33%	33%	33%	47%	51%	13%	20%	34%	30%	83%	7%	8%	36%	63%	30%	53%	6%	9%	16%

Communicator's utility: expectation of any increasing transformation of induced opinions

Focus on two goals of communicators in political scenarios:

- To shift the average opinion towards the communicator's preferred one
- To raise the dispersion of opinions: Iyengar et al., (2012); Iyengar and Westwood (2015); Iyengar et al. (2019); Reiljan et al. (2024); Bäck et al. (2023); Glaeser et al. (2005)

Use of evidence to affect distributions of opinions

It works!, Kuklinski et al. (2000); Bullock (2011); Haaland and Roth (2020); Alesina et al. (2023); Stagnaro and Amsalem (2025)

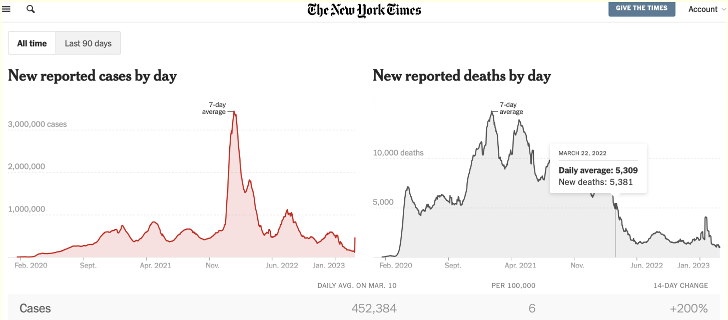
Political leaders, e.g., socioeconomic evidence to *affect opinions on* policy-making quality

Firm managers, e.g., earnings evidence to *affect opinions on* future shareholder returns

Government agencies, e.g., medical evidence to *affect opinions on* the state of a pandemic

How Does Evidence Look Like?

Example – evidence to *affect opinions on* the state of Covid pandemic:



Relevant state $\omega \in \Omega \equiv \{0, 1\}$ and generic prior $\beta \equiv \mathbb{P}(\omega = 1)$ of an audience of Receivers

β NOT deterministic: $\boldsymbol{\beta} \sim F(\beta)$ with $\bar{\beta} = \mathbb{E}[\boldsymbol{\beta}]$ and $\sigma^2 = \mathrm{Var}[\boldsymbol{\beta}]$

Interpretation 1: $F(\beta)$ approximates a frequency distribution (e.g., histogram from survey)

Interpretation 2: $F(\beta)$ theoretical distribution of unknown β

Each prior β revised upon observing **signal** $s \in E \cup \{n\}$:

- $s \in E \subseteq \mathbb{R}$ is a **piece of evidence**
- $s = n$ is **no evidence**

The Game

A Sender chooses an observable **effort** $x \in [0,1] = \mathbb{P}(s \in E) \Rightarrow \mathbb{P}(s = n) = 1 - x$

Effort cost $c : [0,1] \to \mathbb{R}$, twice-differentiable, strictly increasing and convex, with $c(0) = 0$, and Inada conditions $c'(0) = 0$ and $\lim_{x \to 1} c'(x) = +\infty$

Sender (privately) obtains $s \in E \cup \{n\}$ and (publicly) reports $m \in E \cup \{n\}$

Hard evidence: if $s = n \Rightarrow m = n$; if $s \in E \Rightarrow m \in \{s, n\}$

The Game

Sender's **disclosure strategy**: $d : E \cup \{n\} \to \{0,1\}$ with $d(s) = \mathbb{P}(m = s \,|\, s)$ s.t. $d(n) = 1$. Sender's **overall strategy**: (x, d)

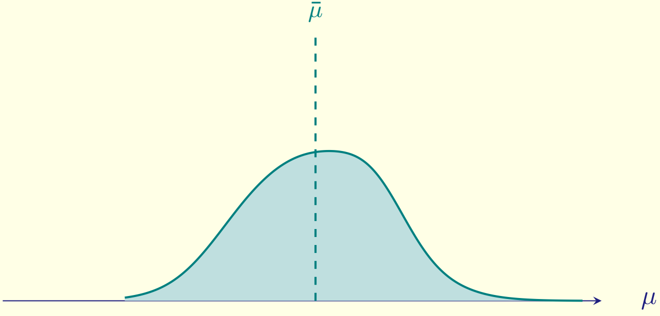
Receiver with prior β updates to $\mu_m^x(\beta) = \mathbb{P}(\omega = 1 \,|\, m; x, d)$. Strategy (x, d) induces **distribution of posteriors** $G_m^x(\mu)$ for $\mu = \mu_m^x$

Sender's interim payoff: $v(G_m^x) = \mathbb{E}[g(\mu_m^x)]$ for any (weakly) increasing function $g : [0,1] \to \mathbb{R}$

Motivation I – Shifting Average Opinion

Normalization consideration: Sender prefers audience to believe that $\omega = 1$

Motivation I: $v(G_m^x) = \mathbb{E}[\mu_m^x]$



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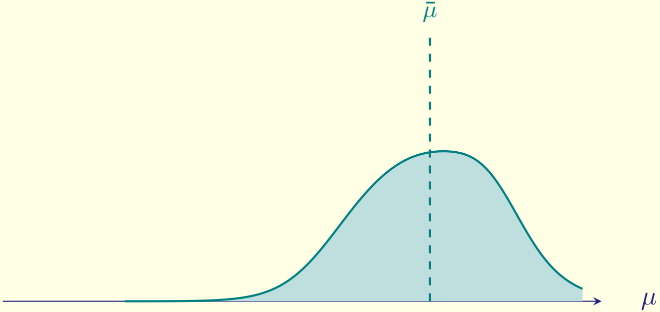
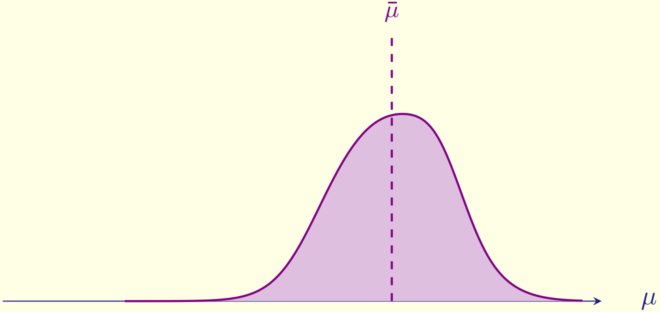


Motivation II – Raising Dispersion of Opinions

Motivation II: $v(G_m^x) = \mathbb{E}[(\mu_m^x)^2]$



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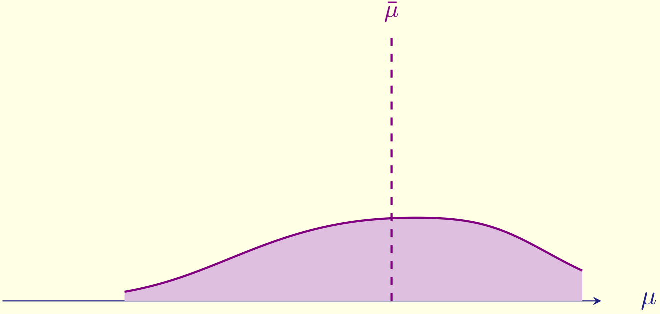

Evidence Provision Technology

$E \subseteq \mathbb{R}$ is a convex set with $\underline{s} \equiv \inf E$ and $\overline{s} \equiv \sup E$

$(E, \mathcal{B}_E)$, measurable space of pieces of evid. with a σ-finite reference measure

For $\omega \in \{0, 1\}$, prob. measure $\mathbb{P}_\omega$ over space $(E, \mathcal{B}_E)$ (absolutely continuous w.r.t. to reference measure $\Rightarrow$ $\mathbb{P}_\omega$ admits Radon-Nikodym derivatives): densities $\pi_\omega(s) > 0$ and cdfs $\Pi_\omega(s) = \int_0^s \pi_\omega(t)\mathrm{d}t$

$\xi \equiv (\mathbb{P}_0, \mathbb{P}_1)$, an evidence provision structure $\equiv$ a Blackwell experiment

Time Line

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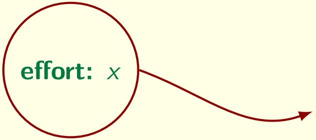
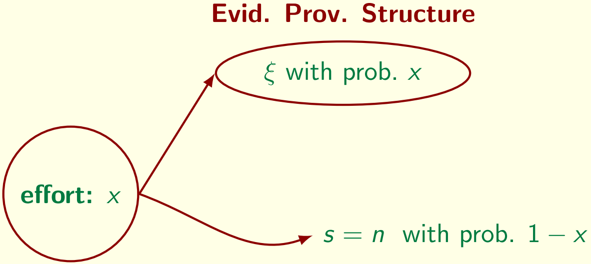


effort: x

$s = n$ with prob. $1 - x$

Time Line



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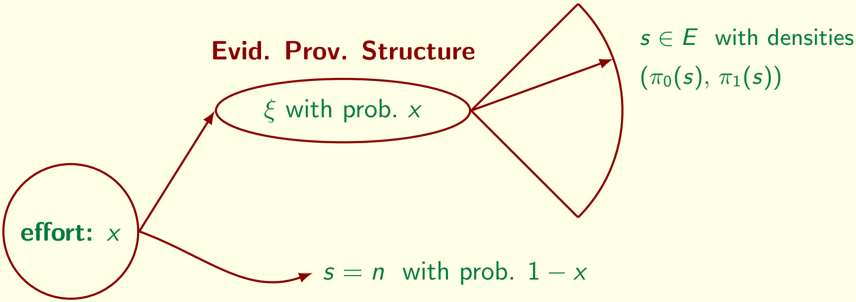
Evid. Prov. Structure
ξ with prob. x
$s \in E$ with densities
$(\pi_0(s), \pi_1(s))$
effort: x
$s = n$ with prob. $1 - x$

Equilibrium

Interim stage: Sender discloses $s \in E$ whenever $v(G_s^x) \geq v(G_n^x)$

Ex ante stage: Sender chooses $x \in [0, 1]$ to maximize:

$$V(x; d) = \mathbb{E}_{\boldsymbol{\beta}}\Big\{(1 - x)v(G_n^x) + x\mathbb{E}_{Q_{\boldsymbol{\beta}}}\big[[1 - d(\boldsymbol{s})]v(G_n^x) + d(\boldsymbol{s})v(G_{\boldsymbol{s}})\big]\Big\} - c(x)$$

$$Q_\beta \equiv \beta\Pi_1 + (1 - \beta)\Pi_0$$

Skepticism and threshold disclosure

$$\mu_s(\beta) = \frac{\lambda(s)\beta}{\lambda(s)\beta + (1-\beta)} \quad \text{with} \quad \lambda(s) \equiv \frac{\pi_1(s)}{\pi_0(s)} \quad \text{for} \quad \pi_0(s) > 0$$

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$$\mu_n^x(\beta) = \frac{\big[(1-x) + x\mathbb{E}_{\Pi_1}[1-d(\boldsymbol{s})]\big]\beta}{\big[(1-x) + x\mathbb{E}_{\Pi_1}[1-d(\boldsymbol{s})]\big]\beta + \big[(1-x) + x\mathbb{E}_{\Pi_0}[1-d(\boldsymbol{s})]\big](1-\beta)}$$

$(1-x) + x\mathbb{E}_{\Pi_\omega}[1-d(\boldsymbol{s})]\big] =$prob. of reporting n given ω under (x, d)

Skepticism and threshold disclosure

$$G_s(\mu) = \mathbb{P}(\mu_s \leq \mu) = F\left(\overbrace{\frac{\mu}{\mu + (1-\mu)\lambda(s)}}^{\beta_s(\mu)}\right)$$

$$G_n^x(\mu) = \mathbb{P}(\mu_n^x \leq \mu) = F\left(\overbrace{\frac{\mu}{\mu + (1-\mu)m(x,d)}}^{\beta_n^x(\mu)}\right)$$

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$$\lambda(s) \equiv \frac{\pi_1(s)}{\pi_0(s)} \quad \text{and} \quad m(x,d) \equiv \frac{(1-x) + x\mathbb{E}_{\Pi_1}[1 - d(\boldsymbol{s})]}{(1-x) + x\mathbb{E}_{\Pi_0}[1 - d(\boldsymbol{s})]}$$

$m(x,d) \rightarrow$ ratio of sizes of concealment under (x,d)

$$\lambda(s) \equiv \frac{\pi_1(s)}{\pi_0(s)} \geq m(x, d) \equiv \frac{(1-x) + x\mathbb{E}_{\Pi_1}[1 - d(\boldsymbol{s})]}{(1-x) + x\mathbb{E}_{\Pi_0}[1 - d(\boldsymbol{s})]}$$

iff $\beta_n^x(\mu) \geq \beta_s(\mu)$ for any $\mu = \beta$

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iff $\quad G_n^x(\beta) = F(\beta_n^x(\beta)) \geq F(\beta_s(\beta)) = G_s(\beta)$

iff $G_s(\beta)$ FOSD $G_n^x(\beta)$ iff $\mathbb{E}[g(\mu_s)] \geq \mathbb{E}[g(\mu_n^x)]$ for monotone g

I. $\mathbb{E}[\mu_s] \geq \mathbb{E}[\mu_n^x]$ and II. $\mathbb{E}[(\mu_s)^2] \geq \mathbb{E}[(\mu_n^x)^2]$

Skepticism and threshold disclosure, what we have shown:

Thm.
d^* discloses piece of evidence $s \in E$ such that $\pi_0(s)\pi_1(s) > 0$ iff

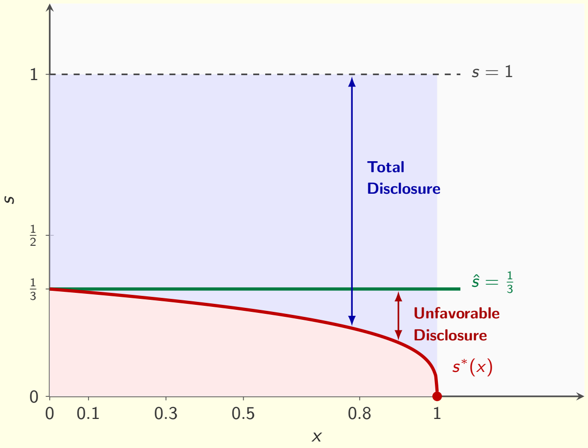
$$\lambda(s) \equiv \frac{\pi_1(s)}{\pi_0(s)} \geq \frac{(1-x) + x\mathbb{E}_{\Pi_1}[1 - d^*(\boldsymbol{s})]}{(1-x) + x\mathbb{E}_{\Pi_0}[1 - d^*(\boldsymbol{s})]}$$

Under regularity conditions, **unique threshold** $s^*(x)$ characterized by:

$$(1-x)[\lambda(s^*(x)) - 1] + x\int_{\underline{s}}^{s^*(x)} \pi_0(s)[\lambda(s^*(x)) - \lambda(s)]\mathrm{d}s = 0$$

Beta Technology

$$\pi_\omega(s) = (\omega+2)(\omega+1)s^\omega(1-s),\ s\in[0,1]\ \Rightarrow \lambda(\hat{s}) = 1 \text{ for } \hat{s} = 1/3$$



Value of evidence acquisition and disclosure to Sender

Equilibrium Sender's Return

$$TR(x; s^*(x)) = v(G_{s^*(x)}) + x\mathbb{E}_{Q_{\bar{\beta}}}\left[v(G_s) - v(G_{s^*(x)}) \,\middle|\, s \geq s^*(x)\right]$$

Equilibrium Sender's Marginal Return:
$MR(x; s^*(x)) \equiv \partial TR(x; s^*(x))/\partial x$

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Equilibrium Sender's Marginal Return:
$MR(x; s^*(x)) \equiv \partial TR(x; s^*(x))/\partial x$

Key implications: $s^*(x)$ is 1. invertible and 2. common for all $v(G_s) = \mathbb{E}[g(\mu_s)]$ with monotone $g \rightarrow MR(s) \equiv MR(y(s); s)$

with $y(s) \equiv (s^*(x))^{-1}$ inverse function of $s^*(x)$

Value of evidence acquisition and disclosure to Sender

Given s, Sender's return related to a measure of likelihood that ξ provides her pieces evidence more favorable than s

Hazard Rate Dominance (HRD) size:

$$\varrho(s) \equiv \frac{1 - \Pi_1(s)}{\pi_1(s)} - \frac{1 - \Pi_0(s)}{\pi_0(s)}$$

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Behavioral considerations, $\varrho(s) \rightarrow$ degree of optimism for Sender over evidence given s (Chew, 1983; Karni and Schmeidler, 1991; Wang and Lehrer, 2024)

Prop. Marginal return to Sender at s :

$$MR(s) = \underbrace{\int_s^{\bar{s}} \left[\psi_0(t) + \mu_t(\bar{\beta})\varrho(t)\right] q_{\bar{\beta}}(t) \frac{\mathrm{d}v(G_t)}{\mathrm{d}t} \mathrm{d}t}_{\text{accumulated flow of benefits}} - \underbrace{\frac{\lambda(s)}{\lambda'(s)} q_{\bar{\beta}}(s) \varrho(s) \frac{\mathrm{d}v(G_s)}{\mathrm{d}s}}_{\text{instant loss}}$$

Shape of $\varrho(t)$ over $[s, \bar{s})$ modulates flow gain and boundary loss

Higher returns for rapidly increasing $\varrho(t)$; lower for rapidly decreasing $\varrho(t)$ with $t \in [s, \bar{s})$

Welfare of the Audience?

Each Receiver chooses $a \in A = [0, 1]$ and obtains $u(a, \omega) = -(a - \omega)^2$

Measuring ex ante welfare of audience (**in equilibrium**),

$$W(\mu_s) \equiv \mathbb{E}_{\boldsymbol{\beta}} \mathbb{E}_{\boldsymbol{\omega}} [u(a^*(\mu_s(\boldsymbol{\beta})), \boldsymbol{\omega})] = -\mathbb{E}_{\boldsymbol{\beta}} \left[\mu_s(\boldsymbol{\beta})[1 - \mu_s(\boldsymbol{\beta})] \right]$$

Derived measure of welfare of audience:

$$W(x; s^*(x)) = x \mathbb{E}_{Q_{\bar{\beta}}} \left[W(\mu_s) \,\middle|\, s \geq s^*(x) \right] + [(1-x) + x Q_{\bar{\beta}}(s^*(x))] W(\mu_{s^*(x)})$$

Welfare of the Audience?

Prop. $W(x; s^*(x))$ increases in $x \Leftrightarrow$ Sender obtains higher return when she wants to raise dispersion of opinions relative to when she wants of shift average opinion

Policy recommendations on evidence acquisition efforts depend on the goal of the communicator

Mandatory minimum efforts benefit audiences if communicators want to raise dispersions; harm audiences if communicators want to shift average opinions

Sender's preferred motivation?

According to $F(\beta)$, initial opinions predominantly biased towards $\omega = 1$ + rapidly increasing $\varrho(t) \Rightarrow$ higher returns when raising dispersion of opinions

According to $F(\beta)$, initial opinions predominantly biased towards $\omega = 1$ + rapidly decreasing $\varrho(t) \Rightarrow$ higher returns when shifting average opinion

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Implications reversed if initial opinions predominantly biased towards $\omega = 0$

Impact of initial average opinion $\bar{\beta}$

$F_2(\cdot)$ FOSD $F_1(\cdot)$, $\Delta\bar{\beta} \equiv \bar{\beta}_2 - \bar{\beta}_1 > 0$

Distributional perturbations with Gateaux differentiability,
$V(s; F) \equiv \mathrm{d}v(G_s|_F)/\mathrm{d}s$ when initial opinions follow the cdf $F(\beta)$,
$\Delta V(s) \equiv V(s; F_2) - V(s; F_1)$:

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$$\delta MR(s;F_2 - F_1) =$$

$$\int_s^{\bar{s}} \frac{\lambda'(t)}{\lambda(t)} \Big[\underbrace{\big(\Pi_0(t) - \Pi_1(t)\big)\Delta\bar{\beta}\, V(t;F_1)}_{\text{survival gain}} + \underbrace{\big(1 - Q_{\bar{\beta}_1}(t)\big)\Delta V(t)}_{\text{net updating effect}} \Big] \mathrm{d}t$$

$$- \varrho(s) \Big[\underbrace{\pi_0(s)\big(\lambda(s) - 1\big)\Delta\bar{\beta}\, V(s;F_1) + q_{\bar{\beta}_1}(s)\Delta V(s)}_{\text{threshold adjustment}} \Big].$$

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Updating effect increases as initial opinions, according to $F_1(\beta)$, are predominantly biased towards $\omega = 0$

Impact of initial dispersion of opinions

Family of (twice-continuously-differentiable) distributions $\{F_r(\beta)\}_{r\in\mathbb{R}}$ with common mean $\bar{\beta}$

Mean-preserving spread: $S_r(y) \equiv \int_0^y \frac{\partial F_r(\beta)}{\partial r} \mathrm{d}\beta \geq 0, \quad \forall y \in [0,1]$

Let $H^k(s; \boldsymbol{\beta})$ be the polynomial function such that $\mathbb{E}_F[H^k(s; \boldsymbol{\beta})] = [\mathrm{d}v(G_s)/\mathrm{d}s]/[\lambda'(s)/\lambda(s)]$:

$H^{\mathrm{I}}(t;\beta) = \mu_t(\beta)(1-\mu_t(\beta))$; $H^{\mathrm{II}}(t;\beta) = 2\mu_t^2(\beta)(1-\mu_t(\beta))$

Then, $H^k_{\beta\beta}(t,\beta) \equiv \frac{\partial^2}{\partial\beta^2} H^k(t,\beta)$ measures the concavity/ convexity in priors of Sender's interim utility

Impact of initial dispersion of opinions

Prop. If $H^k_{\beta\beta}(t,\beta) < 0$ for all $t \in [s^*, \bar{s})$ and all $\beta \in \mathcal{B}$, then the Sender strictly increases her effort $(\mathrm{d}x^*/\mathrm{d}r > 0)$

$$\varrho(s^*) > \int_{s^*}^{\bar{s}} \frac{1 - Q_{\bar{\beta}}(t)}{q_{\bar{\beta}}(s^*)} \frac{\lambda'(t)}{\lambda(t)} \frac{\int_0^1 S_r(\beta) H^k_{\beta\beta}(t,\beta)\mathrm{d}\beta}{\int_0^1 S_r(\beta) H^k_{\beta\beta}(s^*,\beta)\mathrm{d}\beta} \mathrm{d}t$$

If $H^k_{\beta\beta}(t,\beta) > 0$ for all $t \in [s^*, \bar{s})$ and all $\beta \in \mathcal{B}$, then condition reverses.

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If $H^k_{\beta\beta}(t,\beta) > 0$ for all $t \in [s^*, \bar{s})$ and all $\beta \in \mathcal{B}$, then condition reverses.

$H^k_{\beta\beta} < 0 \rightarrow$ accumulated flow of marginal benefits decreases with dispersion (e.g., $H^{\mathrm{I}}_{\beta\beta}(t,\beta) < 0$ when βs are skewed towards $\omega = 1$)

High HRD sizes $\varrho(s^*) \Rightarrow\ \uparrow$ optimism premium $\rightarrow$ shields Sender from concavity of updating

Evidence Disclosure:

Mathews and Postlewaite (1985)

Partial provability:

- Exogenous evidence: Dye (1985); Jung and Kwon (1988); Shin (1994); Acharya, DeMarzo, and Kremer (2011)
- Endogenous information acquisition: Che and Kartik (2009); Kartik, Lee, and Suen (2017)

Uncertainty about Receivers' preferences: Bond and Zeng (2022)

Pricing of acquired evidence: Ali, Lewis, and Wasserman (2023)

Bayesian persuasion with heterogeneous Receivers: Alonso and Camara (2016); Manili (2024)