Affecting Distributions of Opinions with Evidence*

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Abstract

We investigate evidence acquisition and disclosure by a communicator who cares about the distribution of the opinions of an audience. Uncertainty about whether evidence can be obtained allows for selective disclosure. We consider two alternative motivations for the communicator: (i) to shift the average opinion towards her preferred one and (ii) to raise the dispersion of opinions. For general evidence-provision structures, we characterize the unique equilibrium disclosure strategy, which is common for both motivations. The communicator's return in equilibrium is linear in a measure of the extent to which the evidence-provision structure could favor her preferred state conditional on an obtained piece of evidence. This measure reflects the behavioral description of the communicator's degree of optimism over potential evidence. The relation between the welfare of the audience and the acquisition effort depends critically on which of the two motivations yields higher returns to the communicator, an insight useful for policy recommendations. Minimum limits on evidence acquisition benefit audiences only when communicators seek to raise the dispersion of opinions, not when they seek to shift the average opinion. We also analyze how certain features of the evidence-provision structure and of the distribution of initial opinions influence the communicator's return.

Keywords: Distributions of Opinions; Evidence Acquisition; Strategic Disclosure; Sanitization Strategies; Hazard Rates

JEL Classification: C72; D72; D83; D84

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"You see, Mr. President, real leaders don't follow polls. Real leaders change polls." — Governor Chris Christie. National Republican Convention, 2012.

"Figures don't lie, but liars do figure" — Mark Twain

1 Introduction

Managing collective opinions is a central concern in modern democracies and organizations. Over the last decades, politicians, government officials, and organization leaders have become increasingly interested in the distributions of opinions of their audiences. A large body of political science work has documented the ways in which political leaders want to affect the distributions of opinions reflected in campaign polls (Adams et al., 2004; Somer-Topcu, 2009; Lawrence et al., 2011; Pereira, 2019). Policy responsiveness has long been identified as a central motivation for already elected representatives to care about the opinions expressed in surveys and public consultations (Monroe, 1998; Walgrave et al., 2022). In most democracies, government officials elicit opinions through surveys for purposes of anticipatory representation and policy-making (see, for example, the extensive survey by Durovic and Schnatterer 2025).

The most prevalent motivations in many political settings seem to be well captured by the first and second moments of the distributions of opinions. A frequent goal of a communicator (henceforth, Sender) is to shift the *average opinion* of her audience (henceforth, Receivers) towards her preferred one. In some political environments, communicators appear also profoundly interested in raising the *dispersion of opinions* of their audiences. Building on the idea of *affective polarization* (Iyengar et al., 2012; Iyengar and Westwood, 2015; Iyengar et al., 2019; Reiljan et al., 2024; Bäck et al., 2023), political scientists have provided abundant evidence on the presence of motivations to increase polarization. The idea is that parties exploit heightened division to mobilize their supporters by raising the dispersion of opinions on competing parties. Economists have also appealed to the role of core supporters to argue that party leaders prefer to induce higher dispersion of opinions (see, for example, Glaeser et al. 2005).[1] While arguments in favor of the presence of motivations to decrease polarization are mostly based on the plausible benefits from attracting swing voters with moderate opinions, or from improving governance, this does not appear to be an empirically salient feature in most political environments.

[1] Also, from a purely social welfare perspective, some theoretical models even find desirable a certain degree of polarization (see, for example Bernhardt et al. 2009).

In light of such concerns to affect collective opinions, the use of evidence is ubiquitous: leaders and government officials make efforts to gather evidence that they might subsequently disclose. The view that evidence provision can effectively affect distributions of opinions seems a prevalent one (on experimental evidence, see, for example, Kuklinski et al. 2000; Bullock 2011; Alesina et al. 2023; Stagnaro and Amsalem 2025). For instance, Haaland and Roth (2020) found that providing scientific evidence about the impact of immigration on the labor market shifted the average positive opinion on immigration for a representative population sample in the United States.

How do communicators selectively conceal and disclose evidence when they face distributions of opinions? Is evidence acquisition and disclosure more profitable for communicators when shifting average opinions or when raising the dispersion of opinions? What are the welfare implications for the audience? How do features of (i) the available evidence-provision technology and (ii) the initial distributions of opinions influence their incentives to acquire evidence? To address these questions, this paper investigates the mechanisms underlying the decisions of communicators to acquire and disclose evidence when they want to either shift the average opinion (Motivation I) or raise the dispersion of opinions (Motivation II).

As a concrete example, consider a government office that cares about the opinions expressed in surveys about whether policy-making is bad or good.[2] To affect the distribution of opinions, the government office can make evidence-based research about the quality of policy-making (e.g., employment quality, social inequality, crime, public health). Of course, the range of induced opinions is determined by the richness of the existing evidence and by the evidence-provision technology. What evidence disclosure can attain is also constrained by the shape of the initial distribution of opinions. Even upon abundant evidence in support of good policy-making, it is always harder to shift audiences whose opinions are extremely skewed in the opposite direction. In addition, suppose that, depending on the effort exerted by the government office, the available technology can either provide useful evidence on policy-making, or no evidence. Under this uncertainty, if obtaining no evidence cannot be proved, then the government office can opt out of disclosing evidence by reporting not having obtained it. In these situations, the office would be interested in concealing evidence that leads to distributions of opinions conflicting with its goals over the induced distributions.[3]

[2] For instance, the August 2025 Siena Poll for the state of New York provides distributions of opinions (with bad or good as possible outcomes of the poll) on policy-making issues such as the fiscal situation, education policies, energy, health, child care, policing, or immigration (sri.siena.edu).

[3] As another example, consider the opinions captured by surveys in the United States on the severity

1.1 A Preview

In most of this paper, we abstract from actions that the Receivers might take. We also assume that the Sender has preferences directly over the distributions of opinions of the Receivers. We deviate from this reduced-form approach, though, to study the welfare implications for the audience (Section 5).

To enable strategic evidence concealment in equilibrium, we follow the approach of *partial provability*, initiated by a group of influential papers in accounting and finance (Dye, 1985; Jung and Kwon, 1988; Shin, 1994).[4] As in the current paper, Che and Kartik (2009) and Kartik et al. (2017) have previously investigated disclosure by adding information acquisition to partial provability setups. A central highlight is that skepticism incentivizes Senders to exert more effort to successfully obtain evidence. Higher efforts heighten skepticism when unsuccessful research is reported. To counteract this, the Sender wants to disclose some evidence against her goals. The model we investigate produces a unique threshold (for each given acquisition effort) on the set of available pieces of evidence. This threshold separates equilibrium concealment from disclosure and depends smoothly on the acquisition effort. The (interim) incentive-compatibility conditions for the Sender are critically driven by whether the induced distribution of posteriors upon no evidence *first-order stochastically dominates (FOSD)* the one upon evidence. This insight implies a common disclosure threshold for both motivations of the Sender. The Sender discloses more pieces of evidence as her effort increases due to heightened skepticism.

Unlike previous literature, we consider fairly general evidence-provision structures that induce posteriors in a Bayesian Persuasion fashion (Rayo and Segal, 2010; Kamenica and Gentzkow, 2011). We allow for rich spaces of pieces of evidence and a relatively broad class of technologies for evidence provision. The kinds of insights that we derive using rich spaces of pieces of evidence cannot be obtained with binary spaces (or, more generally, with spaces of pieces of evidence whose cardinality simply coincides with that of the existing state space). Hazard rates for pieces of evidence play an essential role in our study of how features of the evidence-provision structure affect the Sender's return.

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of the Covid-19 outbreak. Data from the Pew Research Center (pewresearch.org) shows that the reported opinions (on whether the disease was a major threat to public health) during 2020 changed dramatically as more scientific evidence on the disease was disclosed to the public by the Centers for Disease Control and Prevention. A prevalent view is that opinions on the severity of the disease varied across regions and countries based on the efforts of their respective health agencies to gather evidence and on their disclosure policies.

[4] Considering uncertainty about what evidence can be gathered, and that the Sender cannot prove whether she has obtained evidence, breaks down the classical disclosure unraveling mechanism (Milgrom, 1981; Grossman, 1981; Milgrom and Roberts, 1986).

A mayor drawback of working with a general class of evidence-provision structures is that little can be said directly about the equilibrium effort. More structure would be needed.[5] We work around this problem by leaning on the implication that the equilibrium evidence threshold is a monotone invertible function of the effort. This allows us to express the Sender's return as a function of pieces of evidence along the equilibrium disclosure strategy. Such an approach leads to a key insight on the value of evidence acquisition and disclosure to the Sender. We show that the Sender's (marginal) return is linear in the difference of the inverses of the hazard rates associated with evidence provision, conditional on each value of the state (Proposition 2). This difference formally measures the extent to which the distribution providing evidence conditional on the state preferred by the Sender Hazard Rate Dominates the distribution conditional on the alternative state. Accordingly, we call this difference *Hazard Rate Dominance (HRD)* size. More intuitively, the HRD size evaluated at an obtained piece of evidence captures the likelihood with which the Sender may obtain pieces of evidence more favorable to her preferred state than the already obtained one. In consonance, under behavioral considerations (Karni and Schmeidler, 1991; Wang and Lehrer, 2024), the HRD size measures as well the degree of the Sender's optimism about potential pieces of evidence given an already obtained piece.

Our model sheds light on if and when the Sender obtains comparatively higher returns under each of her two possible motivations. The result that the evidence acquisition threshold is common for both motivations is central to this exercise. The message that emerges from Proposition 3 is that the Sender obtains higher returns when she wants to raise the dispersion of opinions of the audience (rather than shifting the average opinion) if disclosure is able to skew the distribution of opinions towards her preferred state. For situations where disclosure is not able to skew posterior opinions sufficiently towards her preferred state, then higher returns accrue to a Sender that wants to raise the dispersion of opinions if the evidence-provision structure exhibits relatively small HRD sizes. The converse implication also holds: in such situations, the Sender obtains higher returns from wishing to shift the average opinion under relatively high HRD sizes.

Interestingly, we can connect the previous ranking of the Sender's return across her two motivations with a measure of the welfare of the audience. We show (Proposition 4) that our suggested measure of ex ante welfare for the Receivers increases with the Sender's effort if and only if the Sender's return is highest when she wants to raise the dispersion of opinions. In practical environments where limits on evidence acquisition

[5] For applications with particular distributions that describe evidence-provision structures, one would need to use numerical methods or simulations to approach the equilibrium evidence acquisition effort.

efforts can be enforced but actual disclosure cannot be monitored, these insights might offer some guidance for regulators (interested in the welfare of the audience) to design bounds on mandatory efforts.[6] Setting relatively high mandatory minimum efforts would be preferable in situations where communicators are largely motivated by heightening the dispersion of opinions (rather than by shifting the average opinion towards their own). Designing rules that make communicators invest little in evidence acquisition would be preferable in situations where communicators are largely interested in shifting the average opinion towards their preferred opinion.

Our model also allows us to ask how features of the distribution of opinions of the audience influence the Sender's return from her evidence acquisition and selective disclosure. We find that the classical intuition of decreasing returns follows when one considers how far the initial average opinion is from the communicator's goals. The Sender benefits relatively more from shifting an average opinion that is far away from her preferred opinion. Under certain conditions, we find that the slope of the Sender's return with respect to the HRD size becomes steeper in the presence of average initial opinions more distant from the preferred one. In addition, we follow a comparative statics approach to study whether a mean-preserving spread of initial opinions raises the Sender's effort and increases her disclosure. The HRD size associated with the available evidence-provision structure plays also a prominent role in this analysis. Relatively high HRD sizes at a given piece of evidence make the Sender anticipate pieces of evidence more favorable to her preferred opinion (relative to such a piece of evidence). Such evidence would be directly well-aligned with her goal of shifting the average opinion. In addition, such evidence would be also well-aligned with the goal of raising the dispersion of opinions since they would raise the probability weights on the upper tail of the distribution. To complement these forces, a mean-preserving spread would make the opinions of the audience more aligned with the goal of raising the dispersion of opinions. When those conditions are met, a Sender with either of the two motivations wants to raise her acquisition effort and disclosure if initial opinions are more dispersed. We find (Proposition 6) that evidence-provision structures with relatively high HRD sizes make it easier for the Sender to choose higher efforts upon a mean-preserving spread of the initial opinions of the audience.

[6] In the European Union, evidence acquisition efforts during political campaigns are regulated by the European Union Committee's Inquiry with the mission of protecting the general public. For instance, during the 2016 Brexit referendum, this agency required minimum efforts from the communicators of both platforms. Both political platforms gathered and disclosed evidence supporting their views. The watchdog local authorities monitoring compliance with the minimum efforts were the United Kingdom Parliament and the Bingham Center for the Rule of Law.

1.2 Relationships to Literature

As already touched upon, there are obvious similarities with mechanisms explored by a few papers that consider evidence disclosure with partial provability. In a setup with endogenous (covert) evidence acquisition under partial provability, Che and Kartik (2009) elegantly use the insight that higher efforts increase skepticism to show that differences between the priors of a Sender and a Receiver incentivize evidence acquisition. In a model where two competing Senders acquire and disclose evidence to a single Receiver, Kartik et al. (2017) propose a similar disclosure game to ours but with a binary space of pieces of evidence and a very specific symmetric information structure. Their research question is very different from ours as they ask about the impact of competition in evidence provision to a single Receiver. In an extension of their benchmark model, they also consider a more general information structure to discuss the robustness of their main free-riding result on competing efforts between Senders. In that extension, they assume the existence of a threshold-type equilibrium for evidence disclosure. The two preference specifications for our Sender are different from the one in their model as our Sender cares about features of the distribution of opinions. Despite such differences, we formally characterize (Theorem 1) a unique threshold-type of disclosure equilibrium which is reminiscent of the one that they assume in their discussion section.

A paper related to ours is Bond and Zeng (2022). They investigate disclosure by risk-averse firms that are uncertain about the preferences of the audience. Their questions are different from ours as their focus is on firms disclosing exogenously given evidence to people that may have opposing valuations for the disclosed evidence. Concealment of evidence is driven by insurance in the presence of risk-aversion.

Another related line of research investigates how intermediaries design and price information (e.g., Lizzeri 1999). Ali et al. (2022) propose a model where the seller of an asset chooses first whether to purchase evidence from a profit-maximizing intermediary and then whether to disclose it to the potential buyers. Their analysis focuses on the evidence-design and fee-setting problems of the intermediary. As we do, they also pay attention to exogenous (continuous) distribution functions according to which pieces of evidence are obtained. Interestingly, their derivation of equilibrium fees bears some similarities with the incentive-compatibility requirements that determine in our setup whether a piece of evidence is disclosed by our Sender.

Our Sender is not a Bayesian persuader since she has no commitment power over obtained pieces of evidence. Ex ante, she chooses between a fixed signal structure and another experiment that provides no evidence. This Sender must then make interim incentive-compatible choices over the realized signals from both experiments. We might draw certain analytical connections with a group of papers where a privately informed Sender makes ex ante committed choices of signal structures while being subject to interim incentive-compatibility constraints. In Perez-Richet (2014), the Sender learns one of two possible types prior to selecting a committed signal structure. With the goal of signaling that her type is high, the Sender can use the relationship between her type and the signal released by the chosen signal structure. Recent variations to this problem are found in Koessler and Skreta (2023); Zapechelnyuk (2023).

Related to how considering evidence structures richer than what is strictly necessary to convey information about a binary state space can offer novel insights is the work of Ali et al. (2023). They investigate welfare implications from consumers being able to provide evidence about a private characteristic to a monopolist that subsequently price-discriminates. With their rich technology, consumers are able to opt out of disclosing evidence. As in our model, the sort of insights that they can obtain change dramatically when rich, rather than simple, evidence structures are considered. In particular, only rich technologies enable welfare-improving strategies for the consumers.

Finally, there are certain connections in motivations with Alonso and Camara (2016), where a Bayesian persuader faces voters with heterogeneous priors and Manili (2024), where a seller discloses evidence with full provability to buyers with heterogeneous tastes.

The rest of the paper is organized as follows: Section 2 presents the model; Section 8 discusses some of the assumptions; Section 3 presents the main results; Section 5 derives our main result on the welfare of the Receivers; Section 9 concludes.

2 The Model

A *Sender* (she) wants to affect the heterogeneous opinions of an audience $[0,1]$ of *Receivers* (each of them, he) about a *state* $\omega \in \Omega \equiv \{0,1\}$. A generic *prior* is a probability that the state is high, $\beta \equiv P(\omega = 1)$. A distinctive feature of our model is considering a prior β as a realization of a random variable $\boldsymbol{\beta}$, rather than a fixed common prior for all Receivers. The random variable $\boldsymbol{\beta}$ can take values on a set $\mathcal{B} \equiv \text{supp}(\boldsymbol{\beta}) \subseteq [0,1]$ following a cumulative distribution function (cdf) $F(\beta)$. For our main result on the threshold-type equilibrium disclosure strategy, $\mathcal{B}$ can be either a continuum or a finite set. Our results on the Sender's return from evidence acquisition and disclosure rely on considering that $\mathcal{B}$ is a continuum set. The cdf $F(\beta)$ of the Receivers' priors is public information. We use $\bar{\beta} = E[\boldsymbol{\beta}]$ and $\sigma^2 = \text{Var}[\boldsymbol{\beta}]$ to denote the mean and the variance of the Receivers' priors.

This information setup can accommodate two interpretations. On the one hand, with the motivation of capturing collective opinion environments, we would like to regard $F(\beta)$ as a theoretical cdf that approximates an empirical frequency distribution of opinions from a large audience of Receivers. On the other hand, from an analytical viewpoint, we would like to consider as well the interpretation that the Sender is uncertain about the actual priors of her audience but knows that they distribute following $F(\beta)$. Under this second interpretation, our information structure allows for a totally analogous analysis if we consider instead a single Receiver whose priors are unknown to the Sender.[7]

2.1 Evidence Acquisition and Disclosure

The Sender decides on evidence acquisition and disclosure in two stages. In the first stage, she makes a publicly observable effort $x \in [0,1]$ to gather evidence about ω.[8] The *evidence acquisition effort* x determines the probability that an exogenously given *evidence-provision structure* gives the Sender evidence about ω. In particular, following her evidence acquisition effort, the Sender (privately) receives a *signal realization* s. The signal realization s can either be a *piece of hard evidence* $s \in \mathbb{R}$, with probability x, or no evidence, denoted as $s = n$, with probability $1 - x$. We consider *partial provability*: the Sender cannot prove whether she has obtained evidence. The Sender incurs a cost $c(x)$ from choosing effort x, where $c : [0,1] \to \mathbb{R}$ is a twice-differentiable, strictly increasing and convex function, with $c(0) = 0$, that satisfies the Inada conditions $c'(0) = 0$ and $c'(1) \geq 1$.

In the second stage, the Sender receives privately the signal realization $s \in \mathbb{R} \cup \{n\}$ and chooses whether to disclose it (simultaneously) to all the Receivers. Conditional on obtaining $s \in \mathbb{R} \cup \{n\}$, the Sender reports a *message* $m \in \mathbb{R} \cup \{n\}$ with probability $d(s) \equiv P(m = s \,|\, s)$. Nonetheless, we only need to consider pure strategies, $d(s) \in \{0,1\}$, since equilibria take only the form of pure strategies in our setup. If the evidence acquisition effort is unsuccessful, $s = n$, then the Sender can only report precisely n, i.e., $d(n) = 1$. If the evidence acquisition effort is successful, $s \in \mathbb{R}$, then the Sender can either report the exact piece of obtained evidence s or claim that she has obtained no evidence, i.e., $d(s) \in \{0,1\}$. A *disclosure strategy* is a function $d : \mathbb{R} \cup \{n\} \to \{0,1\}$ with the requirement that $d(n) = 1$. An *overall strategy* of the Sender is a pair (x, d).

[7] We thank Tom Palfrey for drawing our attention to this point.

[8] We consider overt effort to avoid the otherwise required technical developments, which are inessential to our main messages. For a version of our setup with covert effort instead, following the fixed-point requirement of equilibrium, the Receivers would correctly predict the unobservable effort (in a self-fulfilling manner in equilibrium). For instance, this is the approach pursued by Che and Kartik (2009) to model covert information acquisition effort.

Given a strategy (x, d), each message $m \in \mathbb{R} \cup \{n\}$ induces posteriors $\mu = \mu_m^x(\beta)$ on Receivers with priors β according to Bayes' rule. Since priors $\boldsymbol{\beta}$ distribute on $\mathcal{B}$ following a cdf $F(\beta)$, posteriors are given by a random variable $\boldsymbol{\mu}$ that distributes on the image set $\mu_m^x(\mathcal{B})$ following an induced cdf $G_m^x(\mu)$. Given our primary interest in investigating environments where a Sender cares about the collective opinions of an audience, we assume that the *interim utility* of the Sender depends on the induced distribution of posteriors $G_m^x(\mu)$ for each given (x, m).

2.2 Sender's Preferences

For each given effort $x \in [0,1]$ and each given reported message $m \in \mathbb{R} \cup \{n\}$, the Sender has *interim preferences* over the induced distributions of posteriors G_m^x which are described by an *interim utility function* $v(G_m^x)$. Following a reduced-form approach, we intentionally abstract from opinions that the Sender might hold about ω. We assume that the Sender is biased in favor of the high state in the sense that prefers Receivers to believe that $\omega = 1$. Our two particular (interim) preference specifications (or motivations) for the Sender are, for each $\boldsymbol{\mu} = \mu_m^x(\boldsymbol{\beta})$ for $x \in [0,1]$ and $m \in \mathbb{R} \cup \{n\}$:

1. Motivation I: average induced posterior: $v(G_m^x) \equiv E[\boldsymbol{\mu}]$;

2. Motivation II: dispersion of induced posteriors: $v(G_m^x) \equiv E[\boldsymbol{\mu}^2]$.

Motivation I fits political settings where communicators care just about the average public opinion. Motivation II fits political settings where communicators want to raise a form of polarization of opinions.

Example 1. —Affecting Distributions of Opinions. This example illustrates some features of our analysis. Consider an audience whose initial opinions β are uniformly distributed on the interval $[0,1]$. The Sender makes an evidence acquisition effort and then receives a piece of evidence $s \in [\underline{s}, \overline{s}] \subseteq \mathbb{R}$ according to a continuous positive density $\pi_\omega(s)$ given that the state is ω. The Sender wants to raise either the average posterior belief $E[\boldsymbol{\mu}_s]$ or the dispersion of posterior beliefs given by the second moment $E[\boldsymbol{\mu}_s^2]$. Suppose that $\pi_0(s) > 0$ for each $s \in [\underline{s}, \overline{s})$ and consider the likelihood ratio function $\lambda(s) = \pi_1(s)/\pi_0(s)$, which indicates how often the piece of evidence s is obtained when the state is $\omega = 1$ (relative to when it is $\omega = 0$). The posterior belief $\mu_s(\beta) \equiv P(\omega = 1 \mid s; \beta)$ of a Receiver with prior β that observes a piece of evidence s, in terms of the likelihood ratio, is then $\mu_s(\beta) = \lambda(s)\beta/[(\lambda(s) - 1)\beta + 1]$. Since μ is a differentiable and strictly increasing function of β, the induced cdf of posteriors $G_s(\mu)$ is

$$G_s(\mu) = P(\mu_s(\boldsymbol{\beta}) \leq \mu) = P\Big(\frac{\lambda(s)\boldsymbol{\beta}}{1+(\lambda(s)-1)\boldsymbol{\beta}} \leq \mu\Big)$$
$$= P\Big(\boldsymbol{\beta} \leq \frac{\mu}{1+(1-\lambda(s))\mu}\Big) = F\Big(\frac{\mu}{1+(1-\lambda(s))\mu}\Big).$$

Since $\boldsymbol{\beta} \sim U[0,1]$, the density $g_s(\mu)$ (associated with the cdf of the induced posterior) can be expressed as a function of the likelihood ratio $\lambda(s)$ as $g_s(\mu) = \big(1+[1-\lambda(s)]\mu\big)^{-2}$. Note that $g_s(\mu)$ decreases in μ if $\lambda(s) < 1$ and increases in μ for $\lambda(s) > 1$. Given a piece of evidence s, the average (across all initial opinions β) of the induced posteriors can be computed as $E[\boldsymbol{\mu}_s] = \int_0^1 \mu_s dG(\mu_s) = \int_0^1 \mu_s(\beta) dF(\beta)$. In particular,

$$E[\boldsymbol{\mu}_s] = \frac{\lambda(s)\big[\lambda(s) - 1 - \ln\lambda(s)\big]}{[\lambda(s)-1]^2}$$

for $\lambda(s) \neq 1$ and $E[\boldsymbol{\mu}_{\hat{s}}] = \bar{\beta} = 0.5$ for the exogenous threshold piece of evidence $\hat{s}$ that satisfies $\lambda(\hat{s}) = 1$. The second moment of the induced posterior is

$$E[\boldsymbol{\mu}_s^2] = \frac{\lambda(s)\big[\lambda^2(s) - 1 - 2\lambda(s)\ln\lambda(s)\big]}{[\lambda(s)-1]^3}$$

for $\lambda(s) \neq 1$ and $E[\boldsymbol{\mu}_{\hat{s}}^2] = \sigma^2 + \bar{\beta}^2 = 1/3$.

Figure 1 depicts how evidence disclosure affects the initial uniform distribution of opinions. Upon observing any piece of evidence s_0 such that $\lambda(s_0) < 1$ the distribution of priors is bent downwards. Any piece of evidence s_1 such that $\lambda(s_1) > 1$ bends the distribution upwards. Both the average induced posterior $E[\boldsymbol{\mu}_s]$ and the dispersion of opinions measured by $E[\boldsymbol{\mu}_s^2]$ increase in s.

We can view the exogenous threshold $\hat{s}$ as a cutoff on which pieces of evidence the Sender would want to disclose and conceal to raise the first or the second moment of the posteriors. However, the Sender also needs to discount the skepticism that her disclosure raises in the Receivers. In the presence of uncertainty about what evidence the Sender can actually gather, Receivers would assess how likely it is that the Sender has actually obtained evidence when reporting unsuccessful research.

We use this example to illustrate that, when the Sender wants to affect the average or the dispersion of induced posteriors, evidence disclosure must follow a threshold pattern. Since the Receivers' skepticism depends on the observed evidence acquisition effort x, the equilibrium threshold s^* depends on x. Then, how is such an endogenous threshold $s^*(x)$

characterized in the presence of general evidence-provision technologies?

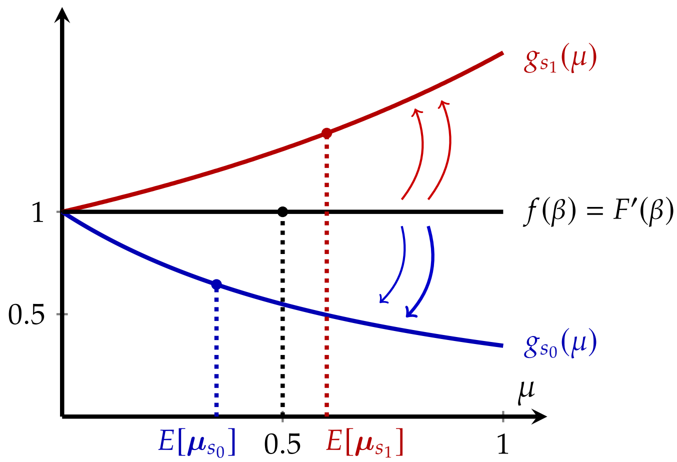

Figure 1 – Affecting the Distribution of Opinions with Evidence.

The initial uniform distribution of opinions is bended downwards with evidence s_0 (in favor of the low state, $\omega = 0$) and upwards with evidence s_1 (in favor of the high state, $\omega = 1$).

2.3 Evidence-Provision Structures

Consider a measurable space of *pieces of evidence* $(\mathbb{R}, \mathcal{B}_{\mathbb{R}})$, where $\mathcal{B}_{\mathbb{R}}$ is the Borel σ-algebra of open sets on $\mathbb{R}$, endowed with a σ-finite reference measure. For each state ω, let P_ω be a probability measure over $(\mathbb{R}, \mathcal{B}_{\mathbb{R}})$. We define an *evidence-provision structure* as a pair $\xi \equiv (P_0, P_1)$ of probability measures in each value ω of the state. Assume that each P_ω is absolutely continuous with respect to the reference measure so that each P_ω admits Radon-Nikodym derivatives or, simply, densities $\pi_\omega(s)$.[9] Let $\Pi_\omega(s)$ denote the cdf that induces the respective measure P_ω for each ω. For each ω, we can consider then the associated random variable $\boldsymbol{s}|\omega$ with respective cdf $\Pi_\omega(s)$. Note that the above definitions do not require that the random variables $\boldsymbol{s}|\omega$ be continuous or discrete random variables. In most parts of the analysis, though, we restrict attention to continuous random variables for tractability (Assumption 2).

The primitives of our framework are then (F, ξ, v), specifying the initial distribution of the Receivers' opinions, the available evidence-provision structure, and the preferences of the Sender over induced distributions of opinions. To ensure that the preferences of the Sender v introduced in Subsection 2.2 are well-defined, and to derive implications on the marginal value from evidence disclosure to the Sender, we restrict attention to

[9] An evidence-provision structure ξ in our setup is thus an experiment in the sense of Blackwell (Blackwell, 1951, 1953).

environments (F, ξ, v) such that the resulting moments of the induced posteriors are finite up to the third moment. In short, our analysis considers evidence disclosure and acquisition games with primitives $\{(F, \xi, v) \mid E[|\boldsymbol{\mu}_m^x(\boldsymbol{\beta})|^3] < \infty \quad \forall x \in [0,1], \forall m \in \mathbb{R} \cup \{n\}\}$.

A crucial concern is to capture real-world experimentation environments where the existing evidence-provision technologies are rich. In most situations in practice this seems to be the case even in the presence of a binary variable of interest. For instance, to assess whether the outcome of policy-making is bad or or good, audiences usually pay attention to evidence on a variety of phenomena such as unemployment, inflation, crime, education, public health, and so on.[10] Most notably, considering rich spaces for the available evidence allows us to study phenomena that, by construction, could not be investigated under a binary space of pieces of evidence. In particular, understanding the extent to which unfavorable pieces to the Sender are disclosed in equilibrium, or the role of evidence-provision technologies in the Sender's return cannot be studied using a binary space of pieces of evidence.[11]

Our general class of evidence-provision structures satisfies the Monotone Likelihood Ratio Property (MLRP), together with a Crossing (C) requirement.

Assumption 1 (Evidence-Provision Structures). An evidence-provision structure $\xi = (P_0, P_1)$ satisfies:

(a) MLRP: for each $s_0, s_1 \in \mathbb{R}$, $s_1 > s_0$ implies $\pi_1(s_1)\pi_0(s_0) \geq \pi_1(s_0)\pi_0(s_1)$;

(b) C: there is some $\hat{s} \in \mathbb{R}$ such that $\pi_1(\hat{s}) \leq \pi_0(\hat{s})$ for each $s < \hat{s}$ and $\pi_1(\hat{s}) \geq \pi_0(\hat{s})$ for each $s > \hat{s}$.

Assumption 1–(a) says that the cdf for evidence in the high state, Π_1, Likelihood-Ratio Dominates the corresponding cdf in the low state, Π_0. In short, we consider that the relative frequency with which evidence favors the high state is weakly increasing in $s \in \mathbb{R}$. Note that all s such that $\pi_1(s) \leq \pi_0(s)$ is evidence in favor of the low state, whereas all s such that $\pi_1(s) \geq \pi_0(s)$ is evidence in favor of the high state. Therefore, Assumption 1–(b) requires any structure to be able to provide evidence both in favor (weakly) both of the low state, $s \in [\underline{s}, \hat{s}]$, and of the high state, $s \in [\hat{s}, \overline{s}]$.[12] Under Assumption 1, the sets of pieces

[10] Even for each of those single issues of interest, there are available pieces of evidence on multiple relevant features. To gain information on unemployment, one can look at pieces of evidence on active job-seekers, social security enrollment, or to pieces of evidence on unemployment according to duration, seasonality, age, or other classifications.

[11] More generally, the message that considering either relatively simple or richer sets of pieces of evidence, or signals, affects crucially the type of implications that can be investigated is pervasive in related work that deals with persuasion or disclosure (see, e.g., Gentzkow and Kamenica 2017; Ali et al. 2023)

[12] While our results do not depend on that sort of crossing conditions, we want to account for situations where the available technology is in fact able to provide evidence in favor of both states.

of evidence which exogenously favor the low and the high state are specified, respectively, by $[\underline{s}, \hat{s}]$ and $[\hat{s}, \overline{s}]$.

For an evidence-provision structure $\xi = (P_0, P_1)$, we are sometimes interested in its associated *likelihood ratio* function $\lambda : \mathbb{R} \to \mathbb{R}$, which maps each piece of evidence $s \in \mathbb{R}$ into a ratio $\lambda(s) \in \mathbb{R}$ that satisfies $\lambda(s)\pi_0(s) = \pi_1(s)$. For pieces of evidence $s \in \mathbb{R}$ such that $\pi_0(s) > 0$, we can set $\lambda(s) \equiv \pi_1(s)/\pi_0(s)$. The likelihood ratio λ associated with an evidence-provision structure ξ intuitively describes the intensity with which ξ discloses evidence in favor of $\omega = 1$ (if $\lambda(s) > 1$) versus in favor of $\omega = 0$ (if $\lambda(s) < 1$).

To guarantee (generic) uniqueness of equilibrium, and to investigate the value of evidence acquisition and disclosure to the Sender, we strengthen further Assumption 1.

Assumption 2 (Regular Evidence-Provision Structures). An evidence-provision structure $\xi = (P_0, P_1)$ satisfies:

(a) for each $\omega \in \Omega$, the random variable $\boldsymbol{s}|\omega$ is a continuous random variable on the measurable space $(\mathbb{R}, \mathcal{B}_{\mathbb{R}})$ of reference;

(b) there exists some $\underline{s}$ and $\overline{s}$, with $-\infty \leq \underline{s} < \overline{s} \leq +\infty$, such that

  (i) each $\pi_\omega(s)$ is positive for $s \in (\underline{s}, \overline{s})$;

  (ii) if $-\infty < \underline{s}$, then $\pi_0(\underline{s}) > 0$ and if $\overline{s} < +\infty$, then $\pi_0(\overline{s}) = 0$ and $\pi_1(\overline{s}) > 0$;

(c) differentiability and strict MLRP: each $\pi_\omega(s)$ is twice-differentiable in $s \in (\underline{s}, \overline{s})$ and $\lambda(s)$ is strictly increasing in $s \in (\underline{s}, \overline{s})$;

(d) SC: there is some $\hat{s} \in (\underline{s}, \overline{s})$ such that $\lambda(\hat{s}) = 1$.

The conditions stated in Assumption 2 imply those in Assumption 1. Assumption 2 imposes minimal conditions that guarantee that the likelihood ratio $\lambda(s)$ is well-defined and twice-differentiable in a real interval $[\underline{s}, \overline{s}] \subseteq \mathbb{R}$. Cases where either $\underline{s} = -\infty$, $\overline{s} = +\infty$, or both, are not ruled out. Therefore, the densities $\pi_\omega(s)$ can have full support on $\mathbb{R}$. Also, $\pi_0(s)$ and $\pi_1(s)$ are, in principle, not constrained to share a common full support. Assumption 2 (b)–(ii) is mostly technical. It guarantees that likelihood ratios and posteriors are well-defined in the extremes $\underline{s}$ and $\overline{s}$ when such extremes are finite. While the main insights of the paper do not depend qualitatively on these technical requirements, they substantially simplify the arguments behind some of our results. Furthermore, Assumption 2 (b)–(ii) leads directly to $\mu_{\overline{s}}(\beta) = \mu^2_{\overline{s}}(\beta) = 1$ for each initial opinion $\beta \in \mathcal{B}$, a normalization-style implication that lowers considerably the notational burden required in the exposition of some of our results.

2.4 Posteriors and Equilibrium Notion

Conditional on the Sender reporting a piece of evidence $m = s \in \mathbb{R}$, a Receiver with priors β forms posteriors using Bayes' rule,

$$\mu_s^x(\beta) = \frac{\pi_1(s)\beta}{\pi_1(s)\beta + \pi_0(s)(1-\beta)} \tag{1}$$

We use $q_\beta(s) \equiv \pi_1(s)\beta + \pi_0(s)(1-\beta)$ to denote the density according to which the evidence-provision structure provides the piece of evidence s when the prior is β. Also, we use $Q_\beta(s)$ to denote the cdf associated with density $q_\beta(s)$. Posteriors conditional on a piece of evidence s being reported do not depend on the effort x. We will henceforth drop the super-index and simply write $\mu_s(\beta)$ instead.

If the Sender reports that her evidence acquisition effort has been unsuccessful, $m = n$, then the Receivers follow Bayes' rule to discount this behavior and to form their posteriors. In particular, Receivers take into account the observed effort x to incorporate a (Bayesian) assessment of whether the Sender has actually obtained no evidence, $s = n$, or has obtained a piece of evidence $s \in \mathbb{R}$ that she is in fact concealing. The expected probability that the available pieces of evidence be concealed under the disclosure strategy d, given ω, is $E_{\boldsymbol{s}\sim\Pi_\omega}[1 - d(\boldsymbol{s})]$. In short, $E_{\boldsymbol{s}\sim\Pi_\omega}[1 - d(\boldsymbol{s})]$ measures the subset of pieces of evidence that is concealed under strategy d when the state is ω. The Receiver assigns probability $(1-x)$ to the Sender having actually obtained no evidence and probability $xE_{\boldsymbol{s}\sim\Pi_\omega}[1 - d(\boldsymbol{s})]$ to the Sender having obtained evidence and concealing it when the state is ω. Then, Bayesian updating requires

$$\mu_n^x(\beta) = \frac{\left[(1-x) + xE_{\boldsymbol{s}\sim\Pi_1}[1 - d(\boldsymbol{s})]\right]\beta}{\left[(1-x) + xE_{\boldsymbol{s}\sim\Pi_1}[1 - d(\boldsymbol{s})]\right]\beta + \left[(1-x) + xE_{\boldsymbol{s}\sim\Pi_0}[1 - d(\boldsymbol{s})]\right](1-\beta)}. \tag{2}$$

The analysis focuses on perfect Bayes-Nash equilibrium of the described game, to which we refer simply as *equilibrium*. In equilibrium, Receivers act as passive players that update their priors according to Eq. (1) and Eq. (2). Equilibrium requires that, for each given acquisition effort x, the Sender chooses a disclosure strategy d^* that maximizes her (second stage) interim utility $v(G_m^x)$, for each obtained signal $s \in \mathbb{R} \cup \{n\}$ and each reported message $m \in \mathbb{R} \cup \{n\}$. In addition, in the first stage, the Sender must choose an evidence acquisition effort x^* that maximizes her ex ante utility. We use $V(x; d)$ to denote the ex ante expected utility of the Sender for an evidence acquisition effort x and a disclosure strategy

d. The particular form of $V(x;d)$ is

$$V(x;d) \equiv E_{\boldsymbol{\beta}\sim F}\Big\{(1-x)v(G_n^x) + xE_{\boldsymbol{s}\sim Q_{\boldsymbol{\beta}}}\big[[1-d(\boldsymbol{s})]v(G_n^x) + d(\boldsymbol{s})v(G_{\boldsymbol{s}})\big]\Big\} - c(x).$$

When considering the ex ante perspective, we will often resort to the density according to which a piece of evidence s is obtained given a prior belief equal to the mean prior, $q_{\bar{\beta}}(s)$, and to its associated cdf, $Q_{\bar{\beta}}(s)$. Note that we can equivalently write $q_{\bar{\beta}}(s) = E_{\boldsymbol{\beta}\sim F}q_{\boldsymbol{\beta}}(s)$ and $Q_{\bar{\beta}}(s) = E_{\boldsymbol{\beta}\sim F}Q_{\boldsymbol{\beta}}(s)$ for each given piece of evidence s. Then, using the linearity of the expectation operator, we find useful to rewrite the above expression as[13]

$$V(x;d) \equiv (1-x)v(G_n^x) + xE_{\boldsymbol{s}\sim Q_{\bar{\beta}}}\big[[1-d(\boldsymbol{s})]v(G_n^x) + d(\boldsymbol{s})v(G_{\boldsymbol{s}})\big] - c(x). \tag{3}$$

The linearity of the expectation operator allows also for the following interpretation of the expression in Eq. (3) above. From her ex ante perspective, the Sender considers an audience that has "on average" an initial opinion $\bar{\beta}$. With Eq. (3), the Sender considers that the evidence-provision structure delivers each piece s following the cdf $Q_{\bar{\beta}}(s)$.

3 Main Results

In this section, we study the form of the Sender's disclosure strategy and return from her equilibrium evidence acquisition and disclosure.

Our main result on the Sender's equilibrium disclosure strategy in the second stage is provided in Theorem 1. Under the class of evidence-provision structures described by Assumption 2, Theorem 1 is useful to characterize a threshold strategy through an integral equation. The Threshold Condition derived in (TC) serves as a useful recipe for a broad range of applications.

Given an overall strategy (x, d) of the Sender, we can compute the (endogenous) *ratio of concealment sizes* for evidence, given $\omega = 1$ relative to given $\omega = 0$, as:

$$m(x,d) \equiv \frac{(1-x) + xE_{\boldsymbol{s}\sim\Pi_1}[1-d(\boldsymbol{s})]}{(1-x) + xE_{\boldsymbol{s}\sim\Pi_0}[1-d(\boldsymbol{s})]}. \tag{4}$$

Theorem 1 establishes that, given an effort level x, the Sender (weakly) prefers to disclose pieces of evidence $s \in \mathbb{R}$ over concealing them whenever the likelihood ratio of the evidence-provision structure, evaluated at such pieces s, is no less than the ratio of concealment sizes. With verifiable evidence, Receivers' skepticism is the force behind

[13] Note that $E_{\boldsymbol{\beta}\sim F}E_{\boldsymbol{s}\sim Q_{\boldsymbol{\beta}}}\big[[1-d(\boldsymbol{s})]v(G_n^x) + d(\boldsymbol{s})v(G_{\boldsymbol{s}})\big] = E_{\boldsymbol{s}\sim Q_{\bar{\beta}}}\big[[1-d(\boldsymbol{s})]v(G_n^x) + d(\boldsymbol{s})v(G_{\boldsymbol{s}})\big]$.

disclosure. While skepticism becomes extreme under full provability—as discussed by Milgrom and Roberts (1986)—, partial provability relaxes the level of skepticism, making strategic concealment possible. This is the case in a setup like ours, with endogenous overt evidence acquisition. Information that the Sender is making high efforts to acquire evidence heightens again Receivers' skepticism even under partial provability. In short, the level of skepticism becomes endogenous and positively related to the Sender's efforts to acquire evidence.

THEOREM 1. Suppose that the Sender has either Motivation I, $v(G_m^x) = E[\boldsymbol{\mu}_m^x]$, or Motivation II, $v(G_m^x) = E[(\boldsymbol{\mu}_m^x)^2]$. Consider a given acquisition effort $x \in [0, 1]$.

(a) Under Assumption 1, the Sender (weakly) prefers in any equilibrium disclosure strategy d^* to disclose a piece of evidence $s \in \mathbb{R}$ such that $\pi_0(s)\pi_1(s) > 0$ (over concealing it) if and only if

$$\lambda(s) \equiv \pi_1(s)/\pi_0(s) \geq m(x, d^*). \tag{5}$$

(b) Under Assumption 2, disclosure in equilibrium follows a threshold-type strategy with a unique threshold $s^*(x) \in (\underline{s}, \hat{s})$ that solves Eq. (5) with equality. All pieces of evidence s above the threshold $s^*(x)$ are disclosed, whereas all pieces below are concealed. The threshold $s^*(x)$ is strictly decreasing in x, with $s^*(0) = \hat{s}$ and $s^*(1) = \underline{s}$.

Consider an evidence-provision structure ξ that satisfies Assumption 2. As shown in the proof of Theorem 1, some algebra from the expression Eq. (5) (with equality) allows us to characterize the equilibrium threshold $s^*(x)$ as the piece of evidence that solves:

$$(1 - x)[\lambda(s^*(x)) - 1] + x \int_{\underline{s}}^{s^*(x)} \pi_0(s)[\lambda(s^*(x)) - \lambda(s)]ds = 0. \tag{TC}$$

Since the unique equilibrium disclosure strategy d^* is characterized under Assumption 2 by the threshold $s^*(x)$, we will henceforth use $(x, s^*(x))$ to describe the overall strategy of the Sender, conditional on optimal evidence disclosure in the second stage of the game.

Shin (1994) coined the term *sanitization strategy* for strategies that disclose only favorable pieces of evidence and conceal all the unfavorable ones. However, it is not obvious what should be an appropriate notion of "favorable pieces of evidence" when Receivers have some information about the evidence that the Sender might be obtaining. Our setup allows for a clear distinction between what are favorable pieces of evidence in an exogenous way (as set by Assumption 1) and what pieces become endogenously favorable for the Sender when information about her efforts permeates. In particular, $\hat{s}$ is the exogenous threshold that separates favorable from unfavorable pieces, whereas $s^*(x)$ gives us the endogenous threshold under effort level x. Of course, the two thresholds coincide in the seminal papers on partial provability without endogenous choice of evidence. Kartik et al. (2017) consider endogenous evidence acquisition and the sort of sanitization strategies proposed by Shin (1994). Their benchmark model assumes a binary space of pieces of evidence. As a result, favorable evidence in the basis of the evidence-provision structure coincides by construction in their model with the endogenously disclosed favorable evidence. Given our relatively general class of evidence-provision structures, we view the result of Theorem 1 as complementing the insights of a number of papers that deal with flexible degrees of skepticism on the Receivers' side and with the notion of sanitization strategies.

Example 2. —Beta Distribution Evidence-Provision Structure.

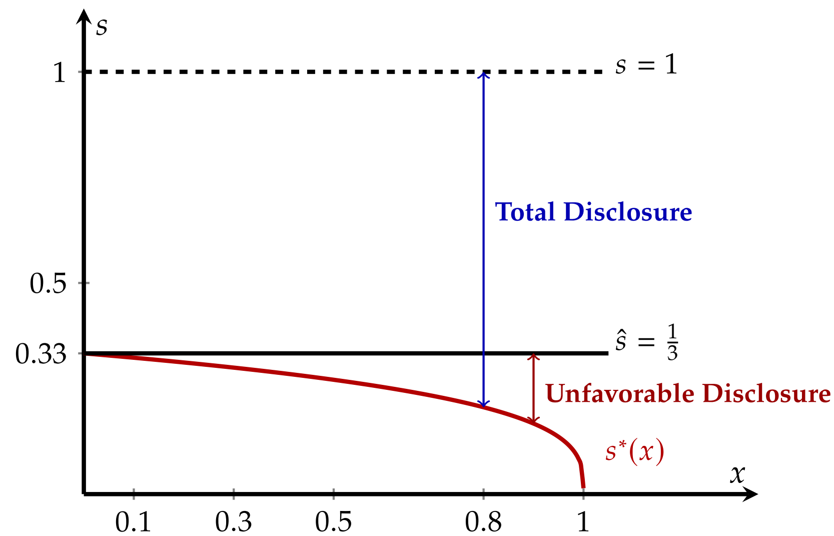

Figure 2 – Equilibrium Disclosure Threshold.

Equilibrium threshold s^* as a function of evidence acquisition effort x when the information structure is described by Beta densities $\pi_\omega(s) = (\omega + 2)(\omega + 1)s^\omega(1 - s)$. The blue arrow depicts the size of total disclosure and the red arrow depicts the size of disclosure of evidence against the preferred opinion of the Sender.

Consider $\underline{s} = 0, \overline{s} = 1$, and suppose that $\boldsymbol{s} \sim \text{Beta}(\omega+1, 2)$ for $\omega \in \{0, 1\}$. The corresponding conditional densities are $\pi_\omega(s) = (\omega + 2)(\omega + 1)s^\omega(1 - s)$. The associated likelihood ratio is $\lambda(s) = \pi_1(s)/\pi_0(s) = 3s$ and the exogenously given threshold $\hat{s}$ (which separates pieces of evidence in favor of each state) is $\hat{s} = 1/3$. The expression in (TC), which characterizes the endogenous threshold $s^*(x)$, takes the form

$$x(s^*)^3 - 3x(s^*)^2 - 3(1-x)s^* + (1-x) = 0.$$

To obtain $s^*(x)$, we need then to solve the above polynomial equation to express s^* as a function of x. For this example with an evidence-provision structure described by Beta densities, Figure 2 depicts (in red) the equilibrium threshold $s^*(x)$ as a function of the exerted effort x.

The form of the Sender's return from evidence acquisition and disclosure follows by plugging the disclosure strategy described by Theorem 1 into the ex ante expected utility derived in Eq. (3).

Corollary 1. Consider Assumption 2 and take a given acquisition effort $x \in [0, 1]$. Then, the Sender's ex ante expected utility, given $s^*(x)$, has the form:

$$V(x; s^*(x)) = v(G_{s^*(x)}) + x \int_{s^*(x)}^{\bar{s}} \left[v(G_s) - v(G_{s^*(x)}) \right] q_{\bar{\beta}}(s) ds - c(x).$$

The value of evidence acquisition and disclosure for the Sender can be decomposed as the sum of her (i) immediate interim utility at the threshold piece of evidence and (ii) cumulative gains from the pieces of evidence that she discloses to the audience. To investigate evidence acquisition efforts, we use the expression in Corollary 1 for the Sender's ex ante expected utility (under the equilibrium disclosure behavior).

To choose her optimal evidence acquisition effort x^* in the first stage, the Sender maximizes her ex ante utility $V(x; s^*(x))$, given the threshold $s^*(x)$ characterized in Theorem 1. To guarantee (generic) uniqueness of equilibrium, we consider evidence-provision structures under Assumption 2. Given the differentiability conditions imposed in Assumption 2, we use $MR(x; s^*(x)) \equiv \partial V(x; s^*(x))/\partial x$ to denote the Sender's return from evidence acquisition effort x conditional on selecting the (unique) threshold piece of evidence $s^*(x)$.

A central insight for our analysis of the Sender's return is the implication of Theorem 1 that $s^*(x)$ is a strictly decreasing function of $x \in [0, 1]$ with $s^*(0) = \hat{s}$ and $s^*(1) = \underline{s}$. Together with this, it turns out also quite useful that such a function $s^*(x)$ is common for both Motivations I and II of the Sender. We exploit these two implications to invert the function $s^*(x)$ and express equivalently the marginal return (conditional on equilibrium disclosure) as $MR(s) = MR(y(s); s)$, where $y(s) \equiv (s^*(x))^{-1}$ identifies the inverse function of $s^*(x)$. Hence, $MR(s)$ specifies the marginal return from an evidence acquisition effort which, in equilibrium, makes the Sender select s as the threshold piece of evidence. As mentioned in the Introduction, this strategy allows us to obtain insights into the incentives of the Sender to acquire evidence for our general class of evidence-provision structures.

Under the mild condition that the marginal return from evidence acquisition is positive to the Sender at $x = 0$, the proposed evidence acquisition and disclosure game has a generically unique interior equilibrium $(x^*, s^*(x^*))$.

Proposition 1. Consider Assumption 2. Suppose that the Sender has any Motivation $k \in \{\mathrm{I}, \mathrm{II}\}$, described by her interim utility function $v(G_m^x)$. Assume that $MR(0, s^*(0)) > 0$. Then, there exists a generically unique interior equilibrium evidence acquisition effort $x^* \in (0,1)$, which is characterized by $MR(x^*; s^*(x^*)) = c'(x^*)$.

A key implication of Proposition 1 is that, for a given evidence acquisition cost function $c(x)$, higher returns $MR(s)$ at an equilibrium threshold $s = s^*(x^*)$ lead to higher evidence acquisition efforts x^* in equilibrium.

We find that the hazard rates associated with the available evidence-provision structure ξ are central to the Sender's marginal return. For pieces of evidence s such that $\pi_0(s) > 0$ and $\pi_1(s) > 0$, we use $IH_\omega(s) \equiv (1 - \Pi_\omega(s))/\pi_\omega(s)$ to denote the inverse of the *hazard rate* of the cdf $\Pi_\omega(s)$. The classical MLRP requirement (which we impose in Assumption 1 and Assumption 2) implies that Π_1 *Hazard Rate Dominates* Π_0. Such a dominance implies that $IH_1(s) \geq IH_0(s)$ for each $s \in [\underline{s}, \overline{s}]$.

In addition, it turns out useful to consider a measure of the degree by which Π_1 Hazard Rate Dominates Π_0, which we label as *Hazard Rate Dominance (HRD) size*. Conditional on the available evidence-provision structure ξ having produced the piece of evidence s, the HRD size $\rho_\xi(s)$ measures how much more likely it is that ξ releases pieces of evidence higher than s given that the state is the one preferred by the Sender, $\omega = 1$, relative to when it is $\omega = 0$.

Definition 1. Under Assumption 2, the *Hazard Rate Dominance (HRD) size* associated with an evidence-provision structure ξ, at a piece of evidence s such that $\pi_0(s) > 0$ and $\pi_1(s) > 0$, is the nonnegative difference

$$\rho_\xi(s) \equiv IH_1(s) - IH_0(s) = \frac{1 - \Pi_1(s)}{\pi_1(s)} - \frac{1 - \Pi_0(s)}{\pi_0(s)}$$

between the inverses of hazard rates conditional on the two possible state realizations.

Formally, $\rho_\xi(s)$ gives us a distance between two probability distributions. In addition, note that the HRD size $\rho_\xi(s)$ coincides with the difference of the *virtual value* functions of a piece of evidence given $\omega = 0$ relative to $\omega = 1$.[14] As first motivated by Bulow and Roberts (1989), in environments where a seller is uncertain about the valuation of the good by a buyer, the virtual value can be interpreted as the marginal value of the good to the buyer. Therefore, using such a classical interpretation of virtual values, $\rho_\xi(s)$ describes a type of (second order) marginal value for the Sender of a marginally higher piece of evidence when the state is $\omega = 0$ relative to when it is $\omega = 1$.

Proposition 2 is instrumental to investigate the Sender's marginal return from evidence acquisition in equilibrium. For each piece of evidence $s \in (\underline{s}, \hat{s})$, the Sender's (marginal) return $MR(s)$ is linear in the HRD size $\rho_\xi(s)$. The derivation of $B(s)$ provided by Proposition 2 is helpful to address a number of questions on the role of the distribution of priors $F(\beta)$ in the Sender's return from evidence acquisition and disclosure.

Proposition 2. Consider Assumption 2 and take any Motivation $k \in \{\text{I}, \text{II}\}$ for the Sender, described by her interim utility function $v(G_m^x)$. Then, given an equilibrium threshold $s \in (\underline{s}, \hat{s})$, the marginal return $MR(s)$ has the following linear form with respect to the HRD size $\rho_\xi(s)$: $MR(s) = A(s) + B(s)\rho_\xi(s)$ for

$$A(s) \equiv \left[1 - q_{\bar{\beta}}(s) IH_1(s)\right]\left[v(G_{\bar{s}}) - v(G_s)\right] \quad \text{and}$$
$$B(s) \equiv (1-\bar{\beta})\pi_0(s)\left[v(G_{\bar{s}}) - v(G_s)\right] - q_{\bar{\beta}}(s)\chi(s),$$

where $\chi(s)$ is a normalization of the marginal interim utility $dv(G_s)/ds$. Moreover, for each $s \in (\underline{s}, \hat{s})$, it follows that $B(s) \geq 0$ if and only if

$$\frac{\int_0^1 F(\beta_s(\mu^{1/(l+1)}))d\mu}{\int_0^1 F(\beta_s(\mu^{1/l}))d\mu} \leq 2 - \mu_s(\bar{\beta}),$$

where $\beta_s \equiv \mu_s^{-1}$ is the inverse function of the posterior at s, and $l \in \{1, 2\}$ is a number such that $l = 1$ for Motivation I and $l = 2$ for Motivation II.

Hazard rates have been extensively used in the design of optimal (i) auctions (Myerson, 1981), (ii) pricing policies (Mussa and Rosen, 1978), and (iii) contracts with moral hazard (Holmström, 1979; Grossman and Hart, 1983; Poblete and Spulber, 2012). Hazard rates are also central to the classical analyses of risk and asset pricing in finance (Jarrow and Turnbull, 1995; Duffie and Singleton, 1999). In auction theory and monopolistic pricing, hazard rates are usually applied to the unknown valuations of the buyers of a good. The

[14] Given a state realization ω, the *virtual value function* evaluated at the piece of evidence s is defined as $s - [1 - \Pi_\omega(s)]/\pi_\omega(s)$.

price that maximizes the expected profit of the seller of the good can be related to the inverse of the hazard rate of the buyers' valuations. In the moral hazard and asset pricing strands, hazard rates are usually applied to random outcomes and returns, which are also influenced by an effort component. In our setup, hazard rates are applied to pieces of evidence, which are also ex ante random. Which particular pieces of evidence are obtained are (ex ante) unknown to the Sender and successfully obtaining them depends on her effort. Our results thus highlight a connection between the Sender's return in terms of inverses of hazard rates and the use of hazard rates, and virtual values, in such classical developments.

Example 3. —Beta Distribution Evidence-Provision Structure. Let us consider again the Beta Distribution technology of Example 2 where $\pi_\omega(s) = (\omega + 2)(\omega + 1)s^\omega(1 - s)$ for $s \in [0, 1]$, associated likelihood ratio $\lambda(s) = \pi_1(s)/\pi_0(s) = 3s$, and exogenously given threshold $\hat{s} = 1/3$. The associated inverse hazard rates are given by

$$IH_0(s) = \frac{1 - s(2 - s)}{2(1 - s)} \quad \text{and} \quad IH_1(s) = \frac{1 - 2(3s - 2s^2)}{6s(1 - s)}.$$

Therefore, the corresponding HRD size takes the form $\rho_\xi(s) = (1 - s)^2/6s$. In this case, $\rho_\xi(s)$ decreases in s, with $\lim_{s\to 0} \rho_\xi(s) = +\infty$ and $\rho_\xi(1/3) = 2/9$. These features depend only on the particular evidence-provision structure ξ and not on the distribution of initial opinions.

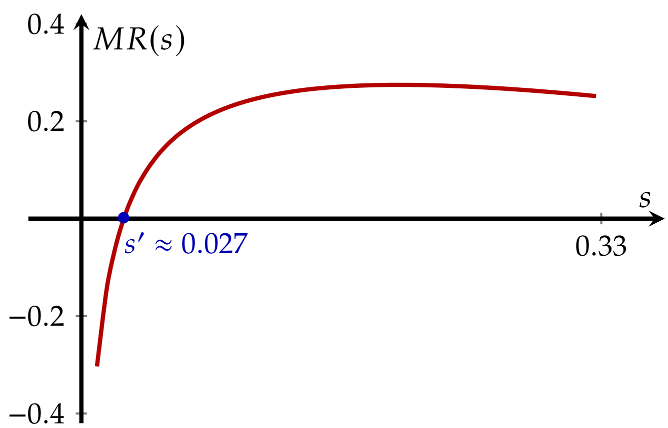

Figure 3 – Shape of $MR(s)$ for Beta Information Structure.

Plot of $MR(s)$ for the evidence-provision structure $\pi_\omega(s) = (\omega + 2)(\omega + 1)s^\omega(1 - s)$.

Consider $\boldsymbol{\beta} \sim U[0, 1]$. In addition, suppose that the Sender wants to raise the mean posterior (Motivation I). Then, it can be verified that

$$B(s) = (1 - s)\left\{E[\overline{\mu}] + \frac{3s}{(3s-1)^3}\left[(3s+1)^2\left[1 - \ln 3s\right] + (3s-1)\ln 3s - 4\right]\right\}.$$

From the developments in Example 1, we can compute $E[\overline{\mu}] = 3(2 - \ln 3)/4 \approx 0.676$. Then, by numerically solving $B(s) = 0$, it can be verified that $B(\tilde{s}) = 0$ for a unique $\tilde{s} \approx 0.2$, with $B(s) > 0$ for each $s \in (0, \tilde{s})$ and $B(s) < 0$ for each $s \in (\tilde{s}, 1/3)$. Furthermore, the Sender's return takes the form

$$MR(s) = \underbrace{\frac{12s^3 - 18s^2 + 1}{2}\left[\frac{(3s-1) - \ln 3s}{(3s-1)^2}\right]}_{A(s)} + \underbrace{\frac{1}{2}\left(\frac{1-s}{3s-1}\right)^3\left[(3s+1)^2\left[1 - \ln 3s\right] + (3s-1)\ln 3s - 4\right]}_{B(s)\rho_\xi(s)}.$$

Using numerical methods, it can be verified that there exists a unique $s' \approx 0.027 \in (0, 1/3)$ such that $MR(s') = 0$ with $MR(s) < 0$ for each $s \in (0, s')$ and $MR(s) > 0$ for each $s \in (s', 1/3)$. Hence, given the negative relationship $s = s^*(x)$, the marginal return for the Sender is higher for lower evidence acquisition efforts and lower for higher effort. In this example, there is thus a negative relationship between $MR(x, s^*(x))$ and x. Above the effort level $x' = y(s')$, the marginal return $MR(x, s^*(x))$ becomes negative. Figure 3 depicts the function $MR(s)$ for $s \in (0, 1/3)$.

For situations where $B(s) > 0$ is guaranteed, it follows that higher values of the HRD size $\rho_\xi(s)$ raise the marginal return of the Sender from evidence disclosure. Nonetheless, even when such a positive relationship between $MR(s)$ and $\rho_\xi(s)$ holds, note that $MR(s)$ is not necessarily increasing in s. How $MR(s)$ changes with s depends largely on the shape of the HRD size $\rho_\xi(s)$, as well as on the particular shape of the terms $A(s)$ and $B(s)$.

Observation. The form of the Sender's return that we have derived can be also interpreted in the light of certain behavioral considerations. The role of hazard rates in the optimistic/pessimistic biases of a decision-maker is also a common topic in the behavioral economics literature. Interestingly, the distance $\rho_\xi(s)$ turns out particularly useful to investigate the size of particular behavioral biases of the Sender when faced with the risk of obtaining a piece of evidence s. The idea is that by replacing the independence axiom by a weaker one, the expected utility theory can be generalized to the weighted utility theory

(Chew, 1983). The weighted utility theory shows the existence of a weight function to represent preferences under risk. Notably, this theory also provides a framework in which optimism/pessimism of a decision maker can be appropriately modeled. By adjusting the definition of Karni and Schmeidler (1991), it follows that, given the ex ante risk that the Sender faces over the pieces of evidence s (associated with the available evidence-provision structure ξ), an optimistic (resp., pessimistic) Sender distorts the probabilities of obtaining favorable pieces of evidence and, in particular, overestimates (resp., underestimates) the chances of obtaining high pieces of evidence. Then, the result of Theorem 2 of Wang and Lehrer (2024) implies that the Sender in our setup always distorts probabilities of obtaining pieces of evidence s in an optimistic direction because, under both Motivations $k \in \{\text{I}, \text{II}\}$, she prefers to be endowed with the cdf $\Pi_1(s)$ rather than $\Pi_0(s)$. In short, our Sender is by construction optimistic when facing the risk of obtaining pieces of evidence favorable to her goals under both Motivations $k \in \{\text{I}, \text{II}\}$. Given this, the implications of Proposition 2 allow us to connect the incentives of the Sender to acquire and disclose evidence to the degree to which she is optimistic about obtaining favorable pieces of evidence. From the result of Proposition 2, it follows that if $B(s) > 0$, then evidence-provision structures that make the Sender more optimistic in a behavioral sense incentivize her relatively more to acquire and disclose evidence.

4 Returns from the Two Motivations

How do the two motivations affect differently the incentives of the Sender to acquire and disclose evidence? The HRD size plays a certain role in which of the two motivations lead to higher (marginal) returns from evidence acquisition and disclosure. In the sequel, we use $MR^k(s)$ to make explicit the Sender's return of evidence acquisition under the respective Motivation $k \in \{\text{I}, \text{II}\}$ (for an equilibrium threshold $s = s^*(x)$). We exploit the insights provided by Proposition 2 to measure the difference $MR^{\text{II}}(s) - MR^{\text{I}}(s)$. Since the function $s^*(x)$ identified in Theorem 1 is common for both Motivations $k \in \{\text{I}, \text{II}\}$, it follows that $MR^{\text{II}}(s) \geq MR^{\text{I}}(s)$ for a given $s \in (\underline{s}, \hat{s})$ if and only if $MR^{\text{II}}(x; s^*(x)) \geq MR^{\text{I}}(x; s^*(x))$ for a given $x \in [0, 1]$, with $s^*(x) = s$.

The first message that emerges from Proposition 3 is that the Sender's return at a given piece of evidence s is higher when she wants to raise the dispersion of opinions (relative to when she wants to shift the average opinion towards her preferred one) when sufficiently high posteriors are induced conditional on s. The second message is that, provided that the induced posteriors are not excessively low, then the Sender benefits comparatively more from raising the dispersion of opinions when using evidence-provision structures with relatively small HRD sizes at the considered piece of evidence s. Conversely, the Sender obtains higher returns when she is motivated by shifting the average posterior under structures with relatively high HRD size $\rho_\xi(s)$.

Proposition 3. Consider Assumption 2. Then, given a common fixed evidence disclosure threshold $s = s^*(x) \in (\underline{s}, \hat{s})$, it follows that $MR^{\mathrm{II}}(s) \geq MR^{\mathrm{I}}(s)$ if and only if

$$\frac{Q_{\bar{\beta}}(s)}{q_{\bar{\beta}}(s)\rho_\xi(s)} \geq \frac{E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)(2\boldsymbol{\mu}_s-1)]}{E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)]}. \tag{6}$$

The ratio in the left-hand-side of condition in Eq. (6) depends only on the evidence-provision structure ξ and on the mean $\bar{\beta}$ of initial opinions. The ratio in the right-hand side depends on the evidence-provision structure ξ and on the entire distribution $F(\beta)$.

We can rewrite the condition in Eq. (6) as

$$q_{\bar{\beta}}(s)E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)(2\boldsymbol{\mu}_s-1)]\rho_\xi(s) \leq Q_{\bar{\beta}}(s)E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)].$$

Note that (i) $q_{\bar{\beta}}(s) < Q_{\bar{\beta}}(s)$ for each given $s > \underline{s}$ since $Q_{\bar{\beta}}(s) = \int_{\underline{s}}^{s} q_{\bar{\beta}}(t)dt$ and $q_{\bar{\beta}}(t) > 0$ for each $t > \underline{s}$ and (ii) $E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)(2\boldsymbol{\mu}_s-1)] \leq E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)]$ since $\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)(2\boldsymbol{\mu}_s-1) \leq \boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)$ almost everywhere (as $\boldsymbol{\mu}_s < 1$ almost everywhere).

Since the underlying state $\boldsymbol{\omega}$ follows a Bernouilli distribution, we have $E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)] = E_{\boldsymbol{\beta}\sim F}E_{\mu_s}[(\boldsymbol{\omega}-\mu_s(\boldsymbol{\beta}))^2]$ and $E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)(2\boldsymbol{\mu}_s-1)] = E_{\boldsymbol{\beta}\sim F}E_{\mu_s}[(\boldsymbol{\omega}-\mu_s(\boldsymbol{\beta}))^3]$. In other words, $E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)]$ gives us the expectation (across priors) of the variance of the state according to the induced posterior $\mu_s(\beta)$ and $E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)(2\boldsymbol{\mu}_s-1)]$ gives us the expectation (across priors) of the skewness of the state according to the induced posterior $\mu_s(\beta)$.

Hence, whether the condition in Eq. (6) is satisfied depends in the first place on the degree of skewness of the induced distribution of opinions, reflected by the sign of the expectation $E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)(2\boldsymbol{\mu}_s-1)]$. First, if the posterior opinions turn out (on average) skewed towards the state least preferred by the Sender, then $E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)(2\boldsymbol{\mu}_s-1)] < 0$ and the condition is directly satisfied. Second, if the posterior opinions turn out (on average) skewed towards the state preferred by the Sender, then $E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)(2\boldsymbol{\mu}_s-1)] > 0$. In such cases, whether the condition is satisfied depends on the HRD size $\rho_\xi(s)$. In these situations, lower HRD sizes $\rho_\xi(s)$ make it easier for the condition Eq. (6) to be satisfied. In particular, $\rho_\xi(s) < 1$ guarantees that $MR^{\mathrm{II}}(s) \geq MR^{\mathrm{I}}(s)$. In contrast, higher HRD sizes $\rho_\xi(s)$ make the condition in Eq. (6) harder to be satisfied.

5 On the Welfare of the Receivers

In this section, we establish a neat relationship between (i) how a measure of the ex ante welfare of the Receivers depends on the evidence acquisition effort x and (ii) which sort of motivation, either I or II, yields higher return to the Sender. We show that if the Sender obtains higher (marginal) return when she wants to increase the dispersion of opinions (relative to when she wants to shift the average opinion), then the proposed measure of welfare for the audience increases with respect to x. Conversely, if the Sender obtains higher marginal returns when her motivation is to shift the average opinion (rather than increasing their dispersion), then the welfare of the audience decreases with respect to x.

To conduct this welfare analysis, we modify our set up slightly, as we discuss in Subsection 8.1. We consider that Receivers choose actions and have preferences over their actions and the state. Specifically, each Receiver faces the quadratic loss decision problem that we describe in Example 5. Each Receiver with priors β chooses an action a from the set $A = [0, 1]$ and has the utility function $u_\beta(a, \omega) = -(a - \omega)^2$. For simplicity, use $a^*_\beta(s)$ to denote the action that maximizes the expected utility of a Receiver with priors β that observes a piece of evidence s. Therefore, a Receiver with priors β optimally chooses $a^*_\beta(s) = 1 \cdot \mu_s(\beta) + 0 \cdot [1 - \mu_s(\beta)] = \mu_s(\beta)$. In short, the Receiver reports his posterior after observing s. Then, the expected utility of a Receiver with priors β that observes a piece of evidence s is

$$\begin{aligned} E_{\mu_s(\beta)}[u_\beta(a^*_\beta(s), \omega)] &= -\left[\mu_s(\beta)(\mu_s(\beta) - 1)^2 + (1 - \mu_s(\beta))(\mu_s(\beta) - 0)^2\right] \\ &= -\mu_s(\beta)[1 - \mu_s(\beta)]. \end{aligned}$$

To measure the welfare of the Receivers, we propose the aggregation of their ex ante expected utilities, given that they optimally choose their actions and that the Sender selects her optimal disclosure threshold. Then, for a piece of evidence s observed by the Receivers, we use

$$W(\mu_s) \equiv E_{\boldsymbol{\beta} \sim F} E_{\mu_s(\boldsymbol{\beta})}[u_{\boldsymbol{\beta}}(a^*(\mu_s(\boldsymbol{\beta})), \omega)] = -E_{\boldsymbol{\beta} \sim F}\left[\mu_s(\boldsymbol{\beta})[1 - \mu_s(\boldsymbol{\beta})]\right]$$

to indicate the aggregation of expected utilities across all Receivers, conditional on their choosing of optimal actions $a^*_\beta(s)$. Nonetheless, recall that the probability that the Receivers actually observe a piece of evidence s depends on the evidence acquisition effort x chosen by the Sender and on the disclosure threshold $s^*(x)$ that she selects. We use $W(x; s^*(x))$ to denote the aggregation of expected utilities across all Receivers, conditional on the equilibrium disclosure threshold $s^*(x)$ and on the acquisition effort x. Assume Assumption 2. Then, in a manner totally analogously to the derivation of the ex ante expected utility of the Sender, we have

$$W(x; s^*(x)) = x \int_{s^*(x)}^{\bar{s}} W(\mu_s) q_{\bar{\beta}}(s) ds + [(1-x) + x Q_{\bar{\beta}}(s^*(x))] W(\mu_{s^*(x)}).$$

The above expression of $W(x; s^*(x))$ specifies our measure of the welfare of the Receivers as a function of the evidence acquisition effort x selected by the Sender.

Interestingly, given an equilibrium threshold $s^*(x)$, the marginal change in the aggregation of expected utilities across all Receivers with respect to the evidence acquisition effort, $dW(x, s^*(x))/dx$, can be expressed in terms of the differences of the (marginal) Sender's return $MR^{\text{k}}(x; s^*(x))$ from her two Motivations $k \in \{\text{I}, \text{II}\}$. In this way, the shape of the ex ante expected utility of the Receivers can be related to which of the two motivations leads to higher returns to the Sender from her evidence acquisition and disclosure.

A prevalent view is that disclosing evidence is always beneficial to an audience that wants to make informed decisions. However, Proposition 4 shows that this is not always the case: when communicators are motivated by shifting the average opinion, higher acquisition effort can harm the audience. The logic is that strategic disclosure may operate in an excessively manipulative manner. Abundant evidence may cause large segments of the audience to drastically revise their views on what the most likely outcome is. From an ex ante perspective, this would harm Receivers that begin with opinions very biased in favor of low state (i.e., Receivers with low values of β). Receivers that begin with opinions already in favor of the high state (i.e., Receivers with high values of β) would not be harmed. Then, using the aggregation of the ex ante expected utilities, the audience would be worse off if a relatively high proportion of Receivers change considerably their opinions.

In practice, regulators have some ability to design enforceable rules for communicators that limit their evidence acquisition efforts but cannot monitor their actual disclosure behaviors. This is the case in environments where regulators can reasonably audit evidence-based research legal agreements or costs. However, identifying whether the efforts made by communicators have in fact been successful seems harder in practice. This would be the case even if evidence could be certified by the authorities. In this sense, regulators usually face the same sort of informational disadvantages as the audiences on what evidence communicators actually gather. In these scenarios, the implications of Proposition 4 could be useful to inform regulatory debates on mandatory efforts to acquire evidence when regulators care about the welfare of the audiences.

Proposition 4. Consider Assumption 2. Let $s^*(x)$ be a fixed common evidence disclosure threshold for each Motivation $k \in \{\text{I}, \text{II}\}$. Then, the marginal change in the aggregation of expected utilities across all Receivers with respect to the evidence acquisition effort takes the form $dW(x; s^*(x))/dx = MR^{\text{II}}(x; s^*(x)) - MR^{\text{I}}(x; s^*(x))$.

Consider a piece of evidence s that is observed by the Receivers. We have seen that it is optimal for each Receiver with priors β to report his posterior $a^*_\beta(s) = \mu_s(\beta)$. Therefore, for that piece of observed evidence s, the aggregation of the maximal expected utilities across all Receivers equals $E[\mu_s^2(\boldsymbol{\beta})] - E[\mu_s(\boldsymbol{\beta})] = E[\boldsymbol{\mu}_s^2] - E[\boldsymbol{\mu}_s]$. The quadratic loss problem that the Receivers face yields an expression for the aggregation of maximal expected utilities that coincides with the difference of the two preference specifications of the Sender. Quadratic loss preferences are nonetheless quite standard in settings under uncertainty where a decision-maker wants to match his action with an unknown variable. Thus, a central planner interested in raising the aggregation of the expected utilities of the Receivers would consider whether the returns that the Sender receives under Motivation II exceed those from Motivation I. Although the particular derivation of the result in Proposition 4 also requires taking into account that the Sender is concealing and disclosing evidence according to the threshold $s^*(x)$, we can still grasp the intuition of how the optimal quadratic losses of the Receivers connect with the difference between the two possible preference specifications of the Sender.

We can establish connections with the results of Section 4. In particular, upon taking $s = s^*(x)$, if the condition stated in Eq. (6) of Proposition 3 holds so that $MR^{\text{II}}(s) > MR^{\text{I}}(s)$ for $s \in (\underline{s}, \hat{s})$, then the aggregation of the expected utilities of the Receivers is increasing in x. Furthermore, recall from the analysis of Section 4 that information structures with lower HRD sizes $\rho_\xi(s)$ make it easier for $MR^{\text{II}}(s) > MR^{\text{I}}(s)$. If the condition stated in Eq. (6) of Proposition 3 is not satisfied, then the aggregation of the expected utilities of the Receivers is decreasing in x.

Of course, the validity of deriving policy recommendations from Proposition 4 rests heavily on restricting attention to scenarios where communicators can in fact be motivated by the two metrics on distributions of opinions investigated in this paper. Such recommendations would not consider environments where communicators are interested in other metrics of the distributions of opinions, such as their kurtosis. As motivated in Introduction, we view the two possible motivations considered in this paper as a good approximation to most common goals of communicators in practice. Our results then recommend regulators to set relatively high bounds on minimal evidence efforts in situations where communicators benefit relatively more from raising the dispersion of opinions. In contrast, in practical situations where communicators obtain higher returns from shifting the average opinion towards their own, the policy recommendation is to enforce relatively low efforts. Perhaps the main message that emerges from the analysis of this section is that knowing which motivation yields higher returns to communicators provides useful guidance for regulators to design enforceable limits on evidence acquisition efforts.

6 Role of the Average Initial Opinion

This section investigates the role of the average initial opinion $\bar{\beta}$ on the slope of marginal return from evidence acquisition and disclosure with respect to the HRD size. We conduct our analysis by comparing two audiences with priors $\boldsymbol{\beta}_i \sim F_i(\beta_i)$ such that $F_2(\beta_2)$ FOSD $F_1(\beta_1)$ and $\bar{\beta}_2 = \bar{\beta}_1 + \delta$, for $\delta \geq 0$ arbitrarily small. Thus, we are considering infinitesimal changes in the mean of priors (by taking $\delta \approx 0$). We use $A_i(s)$ and $B_i(s)$ to denote, respectively, the intercept and the slope of the marginal return identified in Proposition 2. Also, we use $v_i(G_m^x)$ to make explicit that the interim utility function of the Sender is associated with the respective cdf of priors $F_i(\beta_i)$. Then, using the derivation of the terms $A_i(s)$, as obtained in Proposition 2, it can be verified that

$$A_2(s) - A_1(s) = [IH_1(s)q_{\bar{\beta}_1} - 1][v_2(G_s) - v_1(G_s)] + \delta[\pi_0(s) - \pi_1(s)].$$

Therefore, if we consider $\delta \approx 0$, then the difference of terms $A_2(s) - A_1(s)$ approaches $[IH_1(s)q_{\bar{\beta}_1} - 1][v_2(G_s) - v_1(G_s)]$. Nonetheless, as mentioned, our interest in this section is the effect on the slope $B(s)$ of the Sender's return $MR(s)$.

Some additional notation is needed to present our results on the role of the initial average opinion. Note that the inverse function $\beta_s = \mu_s^{-1}$ (which maps posteriors μ into priors $\beta = \beta_s(\mu)$ that induce such posteriors given the piece of evidence s) is (strictly) increasing in μ for each $s \in (\underline{s}, \hat{s})$. Therefore, we can set the nonnegative differences $\Delta_s^{\mathrm{I}}(\mu) \equiv \beta_s(\mu^{1/2}) - \beta_s(\mu)$ and $\Delta_s^{\mathrm{II}}(\mu) \equiv \beta_s(\mu^{1/3}) - \beta_s(\mu^{1/2})$, for the two possible Motivations $k \in \{\mathrm{I}, \mathrm{II}\}$ of the Sender.

Proposition 5. Consider Assumption 2. Consider two different distributions of priors $\boldsymbol{\beta}_i \sim F_i(\beta_i)$, $i = 1, 2$, such that $F_2(\beta_2)$ FOSD $F_1(\beta_1)$, with $\bar{\beta}_2 = \bar{\beta}_1 + \delta$, for an arbitrarily small $\delta \geq 0$. Consider any Motivation $k \in \{\mathrm{I}, \mathrm{II}\}$ for the Sender, described by her interim utility function $v_i(G_m^x)$ under the respective cdf of priors. Consider a given piece of evidence

$s \in (\underline{s}, \hat{s})$. Then, $B_1(s) > B_2(s)$ if and only if

$$\frac{\int_0^1 \left[F_1(\beta_s(\mu^{1/l}) + \Delta_s^k(\mu)) - F_2(\beta_s(\mu^{1/l}) + \Delta_s^k(\mu))\right] d\mu}{\int_0^1 \left[F_1(\beta_s(\mu^{1/l})) - F_2(\beta_s(\mu^{1/l}))\right] d\mu} < 2 - \mu_s(\bar{\beta}_1),$$

for each Motivation $k \in \{\text{I}, \text{II}\}$.

Since the differences Δ_s^k do not depend on the particular distribution $F_i(\beta)$ of priors, the conditions identified in Proposition 5 can be thought of in terms of induced changes in the magnitude of the difference

$$\int_0^1 \left[F_1(\beta_s(\mu^{1/l})) - F_2(\beta_s(\mu^{1/l}))\right] d\mu$$

upon common changes of $\beta = \beta_s(\mu^{1/l})$ (due to changes in μ, given s). The idea is that such changes yield a ratio between cumulative differences, as expressed in the proposition, that lie below the bound $2 - \mu_s(\bar{\beta}_1)$. As an extreme case, the condition given by Proposition 5 is satisfied if the positive difference $F_1(\beta) - F_2(\beta)$ does not increase with β. In addition, since the bound $2 - \mu_s(\bar{\beta}_1)$ decreases with the initial prior $\bar{\beta}_1$, lower means of priors imply more flexible requirements to guarantee that $B_1(s) > B_2(s)$. In short, lower means of initial opinions lead to steeper slopes of the marginal return $MR(s)$ with respect to $\rho_\xi(s)$.

7 Role of the Dispersion of Initial Opinions

Using a parametric approach, this section investigates the role of the dispersion of initial opinions on the Sender's incentives to acquire evidence. We study how the Sender's returns are influenced by infinitesimal changes in the dispersion of the audience's priors following the framework of Diamond and Stiglitz (1974). Accordingly, we use a shift parameter to identify levels of dispersion of priors and consider a parametric version of a mean-preserving spread over the distribution of priors. We use $F_r(\beta)$ to indicate the cdf of the initial opinions when it is parameterized by a certain shift parameter $r \in \mathbb{R}$. By allowing r to vary, we then consider a family $\{F_r(\beta)\}_{r \in \mathbb{R}}$ of plausible distributions of initial opinions. Assume that each cdf $F_r(\beta)$ is twice-continuously-differentiable with respect to β and r.

The framework of Diamond and Stiglitz (1974) rests heavily on restricting attention to cdfs that cross only once. To grasp the idea, consider first only two cdfs, $F_{r_1}(\beta)$ and $F_{r_0}(\beta)$ that share a common mean, and such that: (i) $F_{r_1}(\beta)$ is more dispersed than $F_{r_0}(\beta)$ and (ii)

$F_{r_1}(\beta)$ and $F_{r_0}(\beta)$ cross only once. Since the two cdfs have a common mean, the cumulated difference $\int_0^y [F_{r_1}(\beta) - F_{r_0}(\beta)]d\beta$ measures the size of the mean-preserving spread for any $y \in [0, 1]$. Using this intuition, we can then consider that r_1 and r_0 are sufficiently close to each other, and restrict attention to distributions of priors that have a common mean and cross once, to obtain

$$S_r(y) \equiv \int_0^y \frac{\partial F_r(\beta)}{\partial r} d\beta \geq 0, \quad \text{for } y \in [0, 1]. \tag{7}$$

In Eq. (7) above, $S_r(y)$ gives us the infinitesimal-change version of the size of the mean-preserving spread $\int_0^y [F_{r_1}(\beta) - F_{r_0}(\beta)]d\beta$. For values r_1 and r_0 sufficiently close to each other, the condition in Eq. (7) delivers the implication that $F_{r_1}(\beta)$ puts more probability weights on both tails of the distribution than $F_{r_0}(\beta)$.

Our exercise boils down to measuring how the (marginal) return to effort $MR(s) = MR(y(s); s)$ (with $y(s) = x$ and $s = s^*(x)$) reacts to a marginal change in parameter r under the single-crossing implication in Eq. (7). Consider an equilibrium $(x^*, s^*(x^*))$ of the evidence acquisition and disclosure game, with $x^* \in (0, 1)$. Proposition 1 showed that such an equilibrium exists and is generically unique. Application of the implicit function theorem yields

$$\frac{dx^*}{dr} = -\frac{\partial MR(x^*, s^*(x^*))/\partial r}{\partial MR(x^*, s^*(x^*))/\partial x - c''(x^*)}.$$

Since x^* is maximizing the Sender's ex ante utility, $\partial MR(x^*, s^*(x^*))/\partial x - c''(x^*) < 0$. Thus, the sign of dx/dr is determined by the sign of the sensitivity term $\partial MR(x^*, s^*(x^*))/\partial r$.

Proposition 6 tells us about the direction of the sensitivity of the Sender's return due to an infinitesimal increase in the dispersion of priors. Given any equilibrium threshold $s = s^*(x)$, we have

$$\frac{\partial MR(s)}{\partial r} = -z(G_{s;r})Q_{\bar{\beta}}(s) + q_{\bar{\beta}}(s)\tilde{\chi}(s; r)\rho_{\xi}(s),$$

where $z(G_{s;r})$ is a measure of the degree of convexity of the Sender's interim utility with respect to β and $\tilde{\chi}(s; r)$ is a measure of the degree of concavity/convexity of the marginal change of the Sender's interim utility with respect to the exerted effort x. The latter marginal change is concave in priors when $\tilde{\chi}(s; r) < 0$ (resp., convex when $\tilde{\chi}(s; r) > 0$).[15] The degree of concavity/convexity of the marginal change in the Sender's interim utility due to the exerted effort x (measured by $\tilde{\chi}(s; r)$) has a critical role in the sign of $\partial MR(s)/\partial r$. A central message is that, provided that a rise in acquisition effort does not make the posteriors of the audience excessively skewed towards $\omega = 0$, then an increase in the

[15] The measures $z(G_{s;r})$ and $\tilde{\chi}(s; r)$ are presented formally in the proof of Proposition 6.

dispersion of priors incentivizes the Sender to raise her effort.

Proposition 6. Consider Assumption 2. Let $(x^*, s^*(x^*))$ be a (generically unique) equilibrium of the evidence acquisition and disclosure game under any Motivation $k \in \{\mathrm{I}, \mathrm{II}\}$ of the Sender. Let $l \in \{1, 2\}$ be a number such that $l = 1$ for Motivation I and $l = 2$ for Motivation II. It follows that:

(a) if $\tilde{\chi}(s; r) < 0$, then $\partial MR(s)/\partial r < 0$. Moreover, there exists a certain threshold $\beta^k(s; r) < 1$ such that $\tilde{\chi}(s; r) < 0$ if and only if $F(\beta^k(s; r)) \approx 1$ for each motivation $k \in \{\mathrm{I}, \mathrm{II}\}$;

(b) if $\tilde{\chi}(s; r) > 0$, then $\partial MR(s)/\partial r > 0$ if and only if

$$\frac{-lE[S_r(\beta)[\partial^2 \boldsymbol{\mu}_s^l (1 - \boldsymbol{\mu}_s)/\partial \beta^2]]}{E[S_r(\beta)[\partial^2 \boldsymbol{\mu}_s^l/\partial \beta^2]]} > \frac{Q_{\bar{\beta}}(s)}{q_{\bar{\beta}}(s)\rho_{\xi}(s)},$$

for each Motivation $k \in \{\mathrm{I}, \mathrm{II}\}$.

Proposition 6 (a) considers situations where the marginal change of the Sender's interim utility due to x is a concave function of priors. This implies that the Sender's return decreases as priors become more dispersed. In addition, for any motivation $k \in \{\mathrm{I}, \mathrm{II}\}$, if priors lie below a certain threshold $\beta^k(s; r) < 1$, then such a convexity of the marginal change of the Sender's interim utility in priors is guaranteed. The intuitive message is that if evidence acquisition and disclosure make the opinions of the audience highly skewed towards $\omega = 0$, then the Sender does not find it beneficial to increase her effort when the dispersion of the priors raises.

As for the thresholds $\beta^k(s; r) < 1$ identified in the proposition, it follows that the threshold $\beta^{\mathrm{II}}(s; r)$ (i.e., for Motivation II) is always positive for any evidence-provision structure ξ. However, the threshold $\beta^{\mathrm{I}}(s; r)$ (i.e., for Motivation I) can be negative in some circumstances. In particular, $\beta^{\mathrm{I}}(s; r)$ is negative when $\lambda(s) \geq 1/2$. In such cases, if the disclosed pieces of evidence place high likelihood on $\omega = 1$ for a sufficiently high proportion of Receivers, then the Sender may benefit by increasing her effort when priors become more dispersed. The particular requirement takes the form of the condition stated in Proposition 6 (b). In fact, Proposition 6 (b) considers situations where the marginal change of the Sender's interim utility due to x is a convex function of priors. We provide a sufficient and necessary condition for the marginal return to effort to increase upon an infinitesimal change in the shift parameter r. It follows from the right-hand side of the stated condition, that evidence-provision technologies associated with higher HRD sizes $\rho_{\xi}(s)$ make it easier for the condition to be satisfied. This is a natural implication.

Suppose that the Sender has received a piece of evidence s. Then, relatively high values of the HRD size $\rho_\xi(s)$ indicate that the probability of obtaining pieces higher than s is substantially larger when the state is $\omega = 1$ (compared to when the state is $\omega = 0$). Therefore, relatively high values of $\rho_\xi(s)$ prevent the audience from placing excessive belief weights on $\omega = 0$, This benefits the Sender under both motivations. In such cases, an increase in the dispersion of priors induces a higher level of effort by the Sender.

Why does a convex marginal change of the Sender's interim utility due to x in priors (captured by $\tilde{\chi}(s;r) > 0$) make it easier for the Sender to benefit from increasing her effort upon a rise in the dispersion of priors? Such a convexity indicates that the marginal change the Sender's interim utility (under both motivations) with respect to x becomes small when the degree of skewness of the Receivers' priors towards $\omega = 0$ is relatively high. However, if priors are excessively skewed towards $\omega = 0$ so as to fall below the threshold identified in (b) of the proposition, then the marginal change Sender's interim utility due to x in priors becomes concave instead. Under convexity, such relatively low priors gain more probability weight under the proposed mean-preserving spread of their distribution. In such cases, the Sender has incentives to increase her efforts in response to an increase in the dispersion of priors. We have then that the evidence-provision structure induce posteriors that do not become overly skewed towards $\omega = 0$. This occurs when the degree of convexity of the marginal change of the Sender's interim utility due to x in priors is relatively small. A totally symmetric interpretation in the opposite direction follows when the marginal change of the Sender's interim utility due to x in priors is concave.

8 Discussion of the Assumptions

8.1 Considering Actions by Receivers

As a benchmark, our model has deliberately not considered payoff-relevant actions that Receivers might take. To avoid details that are inessential to the main points of the analysis, we have followed a reduced-form approach to assume that the Sender cares directly about features of the distribution of opinions of the Receivers. Nonetheless, with minor adjustments, our model would be able to capture situations in which Receivers have preferences over their actions and the state. To show this, consider decision problems where each Receiver with priors β chooses an action a from a set A and has a utility function $u_\beta(a, \omega)$. For simplicity, use $a^*_\beta(s)$ to denote an action that maximizes the expected utility of a Receiver with priors β that observes a piece of evidence s. Consider the following three typical examples of such decision problems.

Example 4. —Simple Problem. Consider $A = \{0,1\}$ and $u_\beta(a,\omega) = -(a-\omega)^2$. Then, each of the two actions would be optimal under the corresponding state. In particular, a Receiver with priors β would strictly prefer to choose $a^*_\beta(s) = 1$ if $\mu_s(\beta) > 1/2$, would be indifferent between the two possible actions if $\mu_s(\beta) = 1/2$, and would strictly prefer to choose $a^*_\beta(s) = 0$ if $\mu_s(\beta) < 1/2$.

Example 5. —Quadratic Loss. Consider $A = [0,1]$ and $u_\beta(a,\omega) = -(a-\omega)^2$. Then, a Receiver with priors β would optimally choose $a^*_\beta(s) = 1 \cdot \mu_s(\beta) + 0 \cdot [1-\mu_s(\beta)] = \mu_s(\beta)$.

Example 6. —True Belief Elicitation. Consider $A = \Delta(\Omega)$ and $u_\beta(a,\omega) = (a-\mu(\beta))^2$ for a posterior $\mu(\beta)$ (conditional on some piece of information). Then, a Receiver with priors β would optimally choose $a^*_\beta(s) = \mu_s(\beta)$.

The actions of all three problems in Example 4–Example 6 can be naturally interpreted as the report of a belief. In Example 4, the Receiver would optimally report whether his belief is sufficiently high (above 1/2). In both Example 5 and Example 6, the Receiver would report his actual belief $\mu_s(\beta)$ for each value of the state. Furthermore, since the state space is binary, both the quadratic loss problem (Example 5) and the true belief elicitation problem (Example 6) coincide with Brier's elicitation method (Brier, 1950). In Brier's elicitation method, for each possible value of the state ω, the Receiver wants to announce a belief $a^*_\beta(s)$ that minimizes the (Euclidean) distance to a degenerate posterior that puts probability one on such a state ω. Any of the three utility specifications above can suitably be accommodated by our reduced-form approach. If we considered that the Receivers choose actions as in the previous decision problems, then we would also assume that the Sender wants the Receivers to choose action $a = 1$ regardless of the state.

Interestingly, by adjusting our setup to include actions, as in the three decision problems in Example 4, Example 5, or Example 6, Motivation I gives us the same preference ordering over induced posteriors G^x_m for a Sender that wishes to raise the proportion of Receivers that, respectively, choose actions $a^* = 1$, $a^* = E[\boldsymbol{\omega}]$, or $a^* = \mu_m$. Specifically, the Sender would like to raise, respectively, $G^x_m(1/2)$ (for Example 4) or $G^x_m(\mu^x_m)$ (for Example 5 and Example 6). Any of those goals is equivalent to raising $E[\boldsymbol{\mu}]$ for $\boldsymbol{\mu} = \mu^x_m(\boldsymbol{\beta})$, which corresponds to Motivation I. Of course, Motivation II would give us slightly different considerations as the Sender would wish to induce more dispersed opinions. To investigate how the welfare of the audience depends on the evidence acquisition effort chosen by the Sender, we have resorted in Section 5 to the sort of modifications discussed here and considered that the Receivers can take actions within the quadratic loss decision problem described in Example 5.

8.2 On the Sender's Preferences

As motivated in the Introduction, we have focused on two specific preference specifications that could be of interest in collective opinion and political scenarios. To justify in a more axiomatic manner such particular choices, we now lay out certain desiderata for a preference order over distributions of opinions in a setup like ours. The criteria below consider that the Sender is biased in favor of the high state. Suppose for a moment that there is instead a deterministic prior β. In this case, since the Sender favors $\omega = 1$, a reduced-form preference proposal would require the Sender to prefer higher values of the induced deterministic posterior μ. Given this consideration for a known prior, then the following minimal criteria stand out for an environment where the posterior $\boldsymbol{\mu} = \mu_m^x(\boldsymbol{\beta})$ is instead a random variable.

1. D1: If G_m^x FOSD $G_{m'}^x$, then the Sender should prefer G_m^x over $G_{m'}^x$, for $m, m' \in \mathbb{R} \cup \{n\}$.
2. D2: Provided that $\mu_m^x(\beta)$ is increasing in $m = s \in \mathbb{R}$ for each given $\beta \in \mathcal{B}$, if $s > s'$ for $s, s' \in E$, then the Sender should prefer G_s^x over $G_{s'}^x$.

The two motivations considered in our model comply with desiderata D1 and D2.

8.3 Covert Evidence Acquisition

We have considered overt effort to avoid technical details inessential to our main messages. Suppose instead that the effort x is not observed by the Receivers and that they form a common expectation x^e about the effort exerted by the Sender.[16] With this variation, the source of skepticism of the audience when $m = n$ is reported comes actually from the expected effort x^e, rather than from the actual effort x exerted by the Sender. Under Assumption 2, the Sender will choose in the second stage a unique disclosure threshold $s^*(x^e)$ that solves the integral equation in Eq. (TC) that for any given expected effort $x^e \in [0, 1]$. In the first stage, the Sender will choose an actual effort level $x \in [0, 1]$ in order

[16] Note that the expectation of a Receiver on the effort exerted by the Sender does not depend on his prior about the state. Therefore, we can consider without loss of generality a common expectation over exerted effort.

to maximize her ex ante expected utility

$$V(x; s^*(x), s^*(x^e)) \equiv v(G_{s^*(x^e)}) + x \int_{s^*(x^e)}^{\bar{s}} \left[v(G_s) - v(G_{s^*(x^e)})\right] q_{\bar{\beta}}(s) ds - c(x).$$

Therefore, the Sender will choose an effort level x such that $\partial V(x; s^*(x)), s^*(x^e)/\partial x = c'(x)$. Equilibrium requires that the expectations of the Receivers be correct, $x^e = x$. Therefore, we must have $\partial V(x; s^*(x))/\partial x = c'(x)$ in equilibrium. Under the mild condition $\partial V(0; s^*(0))/\partial x > 0$, Proposition 1 has shown that there exists a generically unique interior $x^* \in (0, 1)$, which that satisfies the above condition for the Sender's optimal effort.

9 Conclusion

Assuming that people share identical opinions on uncertain variables does not capture most real-world environments where communicators face audiences with diverse characteristics or backgrounds. Communicators in political environments and leaders in organizations increasingly attempt to exert some control over the set of diverse opinions of their audiences. In doing so, they often resort to evidence-based communication. In light of these observations, we have set out to investigate what can be learned from considering heterogeneous opinions of audiences that follow known distributions. In practice, reasonable approximations of such distributions are obtained through opinion polls, feedback surveys, or public consultations. The use of such means of belief elicitation has also increased rapidly in recent decades in most democracies and organizations.

When obtaining evidence cannot be proved, communicators can take advantage of selective concealment to pursue their goals regarding the induced opinions over their audiences. In such cases, communicators can exploit the available evidence-provision technologies, and their knowledge of the initial distributions of opinions, to fine-tune their strategies. While such strategies can have little impact on the opinions of some members of their audiences, the views of others may be drastically affected.

We focus on two separate goals for communicators, (i) a purely persuasive motive of shifting the average opinion towards their preferred one and (ii) raising the dispersion of opinions. We relate key features of the available evidence-provision technologies to a communicator's optimal acquisition and disclosure behavior. While using a binary state space, our model allows for a wide class of evidence-provision technologies. To investigate the incentives to acquire evidence, we first characterize the unique threshold-type equilibrium disclosure strategy. Given this characterization, we are then able to connect the returns to a communicator, according to a linear relationship, with the extent to which the distribution that provides evidence conditional on her preferred state Hazard Rate Dominates the one conditional on the alternative state. The practical use of hazard rates establishes connections with classical developments in auctions, contracts, and asset pricing. Features of the distribution of initial opinions of the audience influence the form of such a linear relationship. Our analysis suggests that communicators experience higher returns when facing initial distributions with relatively low averages and high dispersions of opinions.

The hazard rates associated with the available evidence-provision technologies also tell us which of the two goals yields higher returns to communicators in equilibrium. Conditional hazard rates very close to each other make it easier for a rise in the dispersion of opinions to yield higher returns. Sufficiently distant conditional hazard rates make it easier for the purely persuasive goal of shifting the average opinion to yield higher returns.

As to welfare implications, our paper highlights that whether more acquisition of evidence benefits all the members of the audience depends largely on the type of motivation under which the communicator benefits relatively more by acquiring and disclosing such evidence. Our model suggests that, if communicators benefit relatively more under the purely persuasive motive, then more evidence acquisition is overall detrimental for their audiences. This implication follows due to the impact, from an ex ante perspective, on the expected utility of the members of the audience whose initial opinions are very far away from the preferred opinion of the communicator. In contrast, if communicators benefit relatively more under the goal of raising the dispersion of opinions, then more evidence acquisition improves the overall well-being of the audience in equilibrium. Since hazard rates play a prominent role in the differences of returns that accrue in equilibrium under the two goals of communicators, they also provide some regulatory guidance to design limits on mandatory evidence acquisition efforts.

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Appendix. Proofs

PROOF OF THEOREM 1. Consider a given acquisition effort $x \in [0,1]$. Take a given reported message $m \in \mathbb{R} \cup \{n\}$. The posteriors $\mu_m^x(\beta)$, which are specified in Eq. (1) and Eq. (2), are continuously differentiable and strictly increasing in β. Therefore, the inverse functions $\beta_m^x(\mu) \equiv (\mu_m^x)^{-1}$ are also continuously differentiable and strictly increasing in μ. Hence, application of the monotone transformation property for random variables yields

$$G_m^x(\mu) = P(\mu_m^x(\boldsymbol{\beta}) \leq \mu) = P\big(\boldsymbol{\beta} \leq \beta_m^x(\mu)\big) = F(\beta_m^x(\mu)).$$

Suppose that the Sender receives a piece of evidence $s \in \mathbb{R}$. It follows that

(i) if the Sender discloses evidence s, then a Receiver with priors $\beta = \beta_s(\mu)$ observes report $m = s$ and, therefore,

$$G_s(\mu) = F\left(\overbrace{\frac{\pi_0(s)\mu}{\pi_0(s)\mu + \pi_1(s)(1-\mu)}}^{\beta_s(\mu)}\right); \tag{8}$$

(ii) if the Sender conceals evidence s, then a Receiver with priors $\beta = \beta_n^x(\mu)$ observes report $m = n$ and, therefore,

$$G_n^x(\mu) = F\left(\overbrace{\frac{\mu}{\mu + (1-\mu)m(x,d)}}^{\beta_n^x(\mu)}\right), \tag{9}$$

where, using the Bayesian updating rule in Eq. (2), it can be verified that

$$m(x,d) \equiv \frac{(1-x) + xE_{\boldsymbol{s}\sim\Pi_1}[1-d(\boldsymbol{s})]}{(1-x) + xE_{\boldsymbol{s}\sim\Pi_0}[1-d(\boldsymbol{s})]}.$$

(a) Assume Assumption 1. The required arguments rely heavily on the FOSD order over cdfs. Fix an arbitrary prior $\beta = \mu \in (0,1)$ and a piece of evidence $s \in \mathbb{R}$ such that $\pi_0(s)\pi_1(s) > 0$. Then, by comparing the inverse functions $\beta_m^x(\mu) \equiv (\mu_m^x)^{-1}$, for $m \in \mathbb{R} \cup \{n\}$ that appear in Eq. (8) and Eq. (9), we have

$$\begin{aligned} \lambda(s) = \pi_1(s)/\pi_0(s) \geq m(x,d) \;&\Leftrightarrow\; \beta_s^x(\beta) \geq \beta_n(\beta) \\ &\Leftrightarrow\; G_n^x(\beta) = F(\beta_n^x(\beta)) \geq F(\beta_s(\beta)) = G_s(\beta). \end{aligned} \tag{10}$$

Since $\beta = \mu \in (0,1)$ is any arbitrary given prior and F is weakly increasing, then the relationships stated in Eq. (10) hold if and only if G_s (weakly) FOSD G_n^x. Therefore, Eq. (10) is satisfied if and only if $E[\boldsymbol{\mu}_s] \geq E[\boldsymbol{\mu}_n^x]$, that is, if and only if the Sender (weakly) prefers to disclose the piece of evidence s over concealing it under Motivation I.

As to the incentives of the Sender to disclose the piece of evidence s (over concealing it) under Motivation II, the required arguments rely also on the FOSD order. In particular, $E[\boldsymbol{\mu}^2]$ is an increasing function in $\boldsymbol{\mu} \in [0,1]$ and, therefore, all arguments above apply as well.

(b) Assume Assumption 2. Then, the equality $\lambda(s) = m(x,d)$ can be rewritten as

$$\lambda(s) = \frac{(1-x) + xE_{\boldsymbol{s}\sim\Pi_1}[1-d(\boldsymbol{s})]}{(1-x) + xE_{\boldsymbol{s}\sim\Pi_0}[1-d(\boldsymbol{s})]} = \frac{(1-x) + x\int_{\underline{s}}^{\overline{s}}[1-d(t)]\pi_1(t)dt}{(1-x) + x\int_{\underline{s}}^{\overline{s}}[1-d(t)]\pi_0(t)dt}$$

$$\Leftrightarrow (1-x)\,[\lambda(s)-1] + x\int_{\underline{s}}^{\overline{s}}[1-d(t)]\pi_0(t)\big[\lambda(s)-\lambda(t)\big]dt = 0.$$

For a given strategy (x,d), consider the function $\Phi_{(x,d)} : (\underline{s},\overline{s}) \to \mathbb{R}$ specified as

$$\Phi_{(x,d)}(s) \equiv (1-x)\,[\lambda(s)-1] + x\int_{\underline{s}}^{\overline{s}}[1-d(t)]\pi_0(t)\big[\lambda(s)-\lambda(t)\big]dt.$$

Take a disclosure strategy d^* that is part of an equilibrium. From the MLRP property assumed in Assumption 2, it follows that $\Phi_{(x,d^*)}(s)$ is continuous and strictly increasing in $s \in (\underline{s},\overline{s})$. In addition, for the lowest piece of evidence $s = \underline{s}$, we have

$$\Phi_{(x,d^*)}(\underline{s}) = (1-x)\,\big[\lambda(\underline{s})-1\big] + x\int_{\underline{s}}^{\overline{s}}[1-d^*(t)]\pi_0(t)\big[\lambda(\underline{s})-\lambda(t)\big]dt < 0,$$

because Assumption 2 imposes $\lambda(\underline{s}) < 1$. For the highest piece of evidence $s = \overline{s}$, we have

$$\Phi_{(x,d^*)}(\overline{s}) = (1-x)\,[\lambda(\overline{s})-1] + x\int_{\underline{s}}^{\overline{s}}[1-d^*(t)]\pi_0(t)\big[\lambda(\overline{s})-\lambda(t)\big]dt > 0$$

because Assumption 2 imposes $\lambda(\overline{s}) > 1$. Hence, since $\lambda(s)$ is strictly increasing in $s \in (\underline{s},\overline{s})$, the intermediate value theorem guarantees that there exists a unique $s^*(x)$ such that $\Phi_{(x,d^*)}(s^*(x)) = 0$, with $\Phi_{(x,d^*)}(s) \geq 0$ for $s \in [s^*(x),\overline{s}]$ and $\Phi_{(x,d^*)}(s) < 0$ for $s \in [\underline{s},s^*(x))$. Thus, there is a unique $s^*(x) \in (\underline{s},\overline{s})$ such that the Sender discloses each piece of evidence $s \in [s^*(x),\overline{s}]$ and conceals each piece of evidence $s \in [\underline{s},s^*(x))$. Therefore, the disclosure strategy d^* is characterized by the threshold $s^*(x)$ that solves

$$\Phi_{(x,d^*)}(s) = (1-x)\,[\lambda(s)-1] + x\int_{\underline{s}}^{s} \pi_0(t)\big[\lambda(s)-\lambda(t)\big]dt = 0. \tag{11}$$

Note that $\Phi_{(x,d^*)}(\hat{s}) = x\int_{\underline{s}}^{\hat{s}} \pi_0(s)\big[\lambda(\hat{s})-\lambda(s)\big]ds > 0$ and thus $s^*(x) < \hat{s}$.

As to the final statements of the Theorem, note that application of the implicit function theorem on the equality Eq. (11) above at $s^* = s^*(x)$ yields

$$\frac{ds^*}{dx} = \frac{[\lambda(s^*)-1] - \int_{\underline{s}}^{s^*} \pi_0(s)\big[\lambda(s^*)-\lambda(s)\big]ds}{\lambda'(s^*)\big[(1-x)+x\Pi_0(s^*)\big]} < 0, \tag{12}$$

since $\lambda(s^*) < \lambda(\hat{s}) = 1$. In addition, note from Eq. (11) that $s^*(0)$ must satisfy the equation $\lambda(s^*(0)) - 1 = 0$ so that $s^*(0) = \hat{s}$. Likewise, it follows from Eq. (11) that $s^*(1)$ must satisfy the equation

$$\int_{\underline{s}}^{s^*(1)} \pi_0(t)\big[\lambda(s^*(1))-\lambda(t)\big]dt = 0,$$

which is satisfied if and only if $s^*(1) = \underline{s}$. ∎

Proof of Proposition 1. Consider Assumption 2 and take any Motivation $k \in \{\mathrm{I}, \mathrm{II}\}$ for the Sender, described by her interim utility $v(G_m^x)$. Note first that, since $V(x, s^*(x))$ is a continuous function for each $x \in [0,1]$, the extreme value theorem implies that there exists some effort level $x^* \in [0,1]$ that maximizes $V(x, s^*(x))$. In addition, note that, for $x = 1$, we have

$$MR(1, s^*(1)) = \int_{\underline{s}}^{\overline{s}} v(G_s)q_{\bar{\beta}}(s)ds \in (0,1).$$

Secondly, if $MR(0, s^*(0)) > 0$ holds, then the fact that $MR(1, s^*(1)) \in (0,1)$, together with the assumption that the marginal cost of evidence acquisition $c'(x)$ is continuous and strictly increasing in $x \in [0,1]$ with $c'(0) = 0$ and $c'(1) \geq 1$, leads to the conclusion that there exists some $x^* \in (0,1)$ such that $MR(x^*, s^*(x^*)) = c'(x^*)$. To conclude the required arguments, note that if there is more than a single effort $x^* \in (0,1)$ such that $MR(x^*, s^*(x^*)) = c'(x^*)$, then there can be only finite possible such efforts (whose marginal returns equal their respective marginal costs). Each of such efforts would then be a local maximum of the function $V(x, s^*(x))$. Then, to have more than one global maximum of the function $V(x, s^*(x))$, at least two of such efforts must yield a common outcome $V(x, s^*(x))$. Again, there can be only finite possible efforts (1) whose marginal returns equal their respective marginal costs and (2) that yield the common highest value of $V(x, s^*(x))$. If efforts are drawn from the continuous space of efforts using a continuous distribution, then more than one effort satisfying both (1) and (2) is a probability zero event. ∎

Proof of Proposition 2. Consider Assumption 2 and take any Motivation $k \in \{\mathrm{I}, \mathrm{II}\}$ for the Sender, described by her interim utility $v(G_m^x)$. Using the derivation in Corollary 1, it follows that

$$MR(x; s^*(x)) = \frac{dv(G_{s^*})}{ds}\big[(1-x) + xQ_{\bar{\beta}}(s^*)\big]\frac{ds^*}{dx} + \int_{s^*}^{\bar{s}} \big[v(G_s) - v(G_{s^*})\big] q_{\bar{\beta}}(s) ds. \tag{13}$$

As shown in Theorem 1, the Sender's unique disclosure strategy in equilibrium (under Assumption 2), is characterized by $\lambda(s^*(x)) = m(x, d^*)$. This equilibrium requirement directly implies

$$(1-x) + xQ_{\bar{\beta}}(s^*) = \big[(1-x) + x\Pi_0(s^*)\big]\big[1 + \big(\lambda(s^*) - 1\big)\bar{\beta}\big] = \big[(1-x) + x\Pi_0(s^*)\big]\frac{q_{\bar{\beta}}(s^*)}{\pi_0(s^*)}. \tag{14}$$

Then, by combining the expression in Eq. (12) for ds^*/dx, derived in the proof of Theorem 1, with the implication in Eq. (14), it follows that

$$\begin{aligned}
\big[(1-x) + xQ_{\bar{\beta}}(s^*)\big]\frac{ds^*}{dx} &= \frac{q_{\bar{\beta}}(s^*)}{\pi_0(s^*)}\frac{[\lambda(s^*) - 1] - \int_{\underline{s}}^{s^*} \pi_0(s)\big[\lambda(s^*) - \lambda(s)\big]ds}{\lambda'(s^*)} \\
&= \frac{\lambda(s^*)}{\lambda'(s^*)} q_{\bar{\beta}}(s^*)\Big[\frac{1 - \Pi_0(s^*)}{\pi_0(s^*)} - \frac{1 - \Pi_1(s^*)}{\pi_1(s^*)}\Big] \\
&= \frac{\lambda(s^*)}{\lambda'(s^*)} q_{\bar{\beta}}(s^*)\big[IH_0(s^*) - IH_1(s^*)\big].
\end{aligned}$$

Therefore,

$$MR(x; s^*(x)) = -\frac{dv(G_{s^*})}{ds}\frac{\lambda(s^*)}{\lambda'(s^*)} q_{\bar{\beta}}(s^*)\big[IH_1(s^*) - IH_0(s^*)\big] + \int_{s^*}^{\bar{s}} \big[v(G_s) - v(G_{s^*})\big] q_{\bar{\beta}}(s) ds.$$

Fix any evidence disclosure threshold $s = s^*(x) \in (\underline{s}, \hat{s})$ and use the inverse function $x = y(s)$ to rewrite the above expression as

$$MR(s) = MR(y(s), s) = \int_{s}^{\bar{s}} \big[v(G_t) - v(G_s)\big] q_{\bar{\beta}}(t) dt - \frac{dv(G_s)}{ds}\frac{\lambda(s)}{\lambda'(s)} q_{\bar{\beta}}(s)\big[IH_1(s) - IH_0(s)\big].$$

To ease the exposition, consider the positive function $\chi : (\underline{s}, \hat{s}) \to \mathbb{R}_+$ defined as

$\chi(s) \equiv [dv(G_s)/ds]/[\lambda'(s)/\lambda(s)]$. In short, $\chi(s)$ gives us a normalization of the marginal interim utility $dv(G_s)/ds$ by the increase rate $\lambda'(s)/\lambda(s)$ of the likelihood ratio associated with the evidence-provision structure. It can be verified that, $\chi(s) = E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)]$ for Motivation I of the Sender, whereas $\chi(s) = 2E[\boldsymbol{\mu}_s^2(1-\boldsymbol{\mu}_s)]$ for her Motivation II. Then, use the definitions $\chi(s) \equiv [dv(G_s)/ds]/[\lambda'(s)/\lambda(s)]$ and $\rho_\xi(s) = IH_1(s) - IH_0(s)$, to obtain

$$MR(s) = \int_s^{\bar{s}} \big[v(G_t) - v(G_s)\big] q_{\bar{\beta}}(t)dt - \chi(s) q_{\bar{\beta}}(s)\rho_\xi(s). \tag{15}$$

Now, integrate by parts the term $\int_s^{\bar{s}} v(G_t)q_{\bar{\beta}}(t)dt$ [use $u = v(G_t)$ and $dv = q_{\bar{\beta}}(t)dt$] to obtain

$$\begin{aligned}
\int_s^{\bar{s}} v(G_t)q_{\bar{\beta}}(t)dt &= v(G_t)\int q_{\bar{\beta}}(t)dt\Big|_{t=s}^{t=\bar{s}} - \int_s^{\bar{s}}\int_s^{\bar{s}} q_{\bar{\beta}}(t)dt\frac{dv(G_t)}{dt}dt \\
&= v(G_{\bar{s}})Q_{\bar{\beta}}(\bar{s}) - v(G_s)Q_{\bar{\beta}}(s) - \big[1 - Q_{\bar{\beta}}(s)\big]\big[v(G_{\bar{s}}) - v(G_s)\big] \\
&= v(G_{\bar{s}})Q_{\bar{\beta}}(s) + v(G_s)\big[1 - 2Q_{\bar{\beta}}(s)\big].
\end{aligned}$$

Combine the expression derived above with

$$\int_s^{\bar{s}} v(G_s)q_{\bar{\beta}}(t)dt = v(G_s)\big[1 - Q_{\bar{\beta}}(s)\big]$$

to obtain

$$\int_s^{\bar{s}} \big[v(G_t) - v(G_s)\big]q_{\bar{\beta}}(t)dt = \big[v(G_{\bar{s}}) - v(G_s)\big]Q_{\bar{\beta}}(s).$$

Use the definition of the inverses of the hazard rates $IH_\omega(s)$ to obtain

$$\begin{aligned}
Q_{\bar{\beta}}(s) &= \bar{\beta}\Pi_1(s) + (1-\bar{\beta})\Pi_0(s) \\
&= \bar{\beta}\big[1 - \pi_1(s)IH_1(s)\big] + (1-\bar{\beta})\big[1 - \pi_0(s)IH_0(s)\big] \\
&= 1 - \bar{\beta}\pi_1(s)IH_1(s) - (1-\bar{\beta})\pi_0(s)IH_0(s) \\
&= \big[1 - q_{\bar{\beta}}(s)IH_1(s)\big] + (1-\bar{\beta})\pi_0(s)\big[IH_1(s) - IH_0(s)\big],
\end{aligned}$$

so that

$$\int_s^{\bar{s}} \big[[v(G_t) - v(G_s)\big]q_{\bar{\beta}}(t)dt = \big[v(G_{\bar{s}}) - v(G_s)\big]\Big[\big[1 - q_{\bar{\beta}}(s)IH_1(s)\big] + (1-\bar{\beta})\pi_0(s)\rho_\xi(s)\Big].$$

Finally, by plugging the expression derived above into the marginal return obtained in

Eq. (15) and by readjusting terms, it follows that

$$MR(s) = \left[v(G_{\bar{s}}) - v(G_s)\right]\left[1 - q_{\bar{\beta}}(s)IH_1(s)\right] + \left[(1-\bar{\beta})\pi_0(s)\left[v(G_{\bar{s}}) - v(G_s)\right] - q_{\bar{\beta}}(s)\chi(s)\right]\rho_{\xi}(s).$$

By setting $A(s) = \left[v(G_{\bar{s}}) - v(G_s)\right]\left[1 - q_{\bar{\beta}}(s)IH_1(s)\right]$ and $B(s) = (1-\bar{\beta})\pi_0(s)\left[v(G_{\bar{s}}) - v(G_s)\right] - q_{\bar{\beta}}(s)\chi(s)$, the above expression is rewritten as stated in the Proposition.

We turn to show that, for evidence-provision structures with $\pi_0(\bar{s}) = 0$ and $\pi_1(\bar{s}) > 0$ when $\bar{s} < +\infty$, for each $s \in (\underline{s}, \hat{s})$ we have: $B(s) \geq 0$ if and only if

$$\frac{\int_0^1 F(\beta_s(\mu^{1/(l+1)}))d\mu}{\int_0^1 F(\beta_s(\mu^{1/l}))d\mu} \leq 2 - \mu_s(\bar{\beta}).$$

Use first the fact that $\boldsymbol{\mu}_s$ is a nonnegative random variable, together with the change of random variables formula, to derive

$$E[\boldsymbol{\mu}_s] = \int_0^1 P(\boldsymbol{\mu}_s > \mu)d\mu = \int_0^1 \left[1 - F(\beta_s(\mu))\right]d\mu.$$

In a totally analogous way, derive the following moments of the random variable $\boldsymbol{\mu}_s$:

$$E[\boldsymbol{\mu}_s^2] = \int_0^1 P(\boldsymbol{\mu}_s^2 > \mu)d\mu = \int_0^1 \left[1 - F(\beta_s(\mu^{1/2}))\right]d\mu,$$

and

$$E[\boldsymbol{\mu}_s^3] = \int_0^1 P(\boldsymbol{\mu}_s^3 > \mu)d\mu = \int_0^1 \left[1 - F(\beta_s(\mu^{1/3}))\right]d\mu,$$

where recall that $\beta = \beta_s(\mu) = (\mu_s)^{-1}$ is the (monotone) function that maps posteriors μ into associated priors β, given the piece of evidence s. Under $\pi_0(s) > 0$, the inverse function

$$\beta_s(\mu) = \frac{\mu}{\lambda(s) + [1 - \lambda(s)]\mu}$$

is (strictly) increasing in μ for each $s \in (\underline{s}, \hat{s})$ since $\lambda(s) \in (0, 1)$ for $s \in (\underline{s}, \hat{s})$. It follows that $\beta_s(\mu) < \beta_s(\mu^{1/2}) < \beta_s(\mu^{1/3})$. Furthermore, the shape of the function $\beta_s(\mu)$ does not depend on the cdf $F(\beta)$.

For expositional ease, we associate the number $l = 1$ for Motivation I and the number $l = 2$ for Motivation II. Then, using the expressions for $E[\boldsymbol{\mu}_s]$ and $E[\boldsymbol{\mu}_s^{l+1}]$, for $l \in \{1, 2\}$, obtained above, together with

$$B(s) = (1-\bar{\beta})\pi_0(s)\big[v(G_{\bar{s}}) - v(G_s)\big] - q_{\bar{\beta}}(s)\chi(s),$$

we can rewrite such a term as

$$B(s) = (1-\bar{\beta})\pi_0(s)\int_0^1 F(\beta_s(\mu^{1/l}))d\mu - q_{\bar{\beta}}(s)\int_0^1 \big[F(\beta_s(\mu^{1/(l+1)})) - F(\beta_s(\mu^{1/l}))\big]d\mu$$

for each Motivation captured by $l \in \{1,2\}$. By dividing the expression above over $q_{\bar{\beta}}(s)$, it follows then that $B(s) \geq 0$ if and only if

$$\frac{\int_0^1 F(\beta_s(\mu^{1/(l+1)}))d\mu}{\int_0^1 F(\beta_s(\mu^{1/l}))d\mu} \leq 2 - \mu_s(\bar{\beta}),$$

as stated. ∎

Proof of Proposition 3. Consider Assumption 2. Use $v^k(G_s)$ to make explicit that the interim utility function of the Sender corresponds to Motivation $k \in \{\text{I}, \text{II}\}$. Likewise, use $\chi^k(s)$ to make explicit that such a function corresponds to Motivation $k \in \{\text{I}, \text{II}\}$. It follows from Proposition 2 that $MR^{\text{II}}(s) \geq MR^{\text{I}}(s)$ if and only if

$$MR^{\text{II}}(s) - MR^{\text{I}}(s) = Q_{\bar{\beta}}(s)\big[v^{\text{I}}(G_s) - v^{\text{II}}(G_s)\big] - q_{\bar{\beta}}(s)\rho_{\xi}(s)\big[\chi^{\text{II}}(s) - \chi^{\text{I}}(s)\big] \geq 0.$$

Since $v^{\text{I}}(G_s) = E[\boldsymbol{\mu}_s]$, $v^{\text{II}}(G_s) = E[\boldsymbol{\mu}_s^2]$, $\chi^{\text{I}}(s) = E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)]$, and $\chi^{\text{II}}(s) = 2E[\boldsymbol{\mu}_s^2(1-\boldsymbol{\mu}_s)]$, it then follows that $MR^{\text{II}}(s) \geq MR^{\text{I}}(s)$ if and only if

$$\frac{Q_{\bar{\beta}}(s)}{q_{\bar{\beta}}(s)\rho_{\xi}(s)} \geq \frac{E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)(2\boldsymbol{\mu}_s - 1)]}{E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)]},$$

as stated. ∎

Proof of Proposition 4. Consider Assumption 2. Let $s \equiv s^*(x)$ be a fixed common evidence disclosure threshold for each Motivation $k \in \{\text{I}, \text{II}\}$. Then, taking $s = s^*(x)$, the marginal change in the aggregation of expected utilities across all Receivers with respect to the evidence acquisition effort is given by

$$\frac{dW(x; s^*(x))}{dx} = \int_s^{\bar{s}} [W(\mu_t) - W(\mu_s)]q_{\bar{\beta}}(t)dt + [(1-x) + xQ_{\bar{\beta}}(s)]\frac{\partial W(\mu_s)}{\partial x}.$$

Use $W(\mu_s) = -E_{\boldsymbol{\beta}\sim F}\big[\mu_s(\boldsymbol{\beta})[1-\mu_s(\boldsymbol{\beta})]\big] = -E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)]$ to rewrite the above expression for $dW(x;s^*(x))/dx$ as

$$\int_s^{\bar{s}} E[\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)-\boldsymbol{\mu}_t(1-\boldsymbol{\mu}_t)]q_{\bar{\beta}}(t)dt + \big[(1-x)+xQ_{\bar{\beta}}(s)\big]\frac{\partial E[\boldsymbol{\mu}_s(\boldsymbol{\mu}_s-1)]}{\partial x}$$
$$= \int_s^{\bar{s}} \big[E[\boldsymbol{\mu}_t^2]-E[\boldsymbol{\mu}_t]+E[\boldsymbol{\mu}_s]-E[\boldsymbol{\mu}_s^2]\big]q_{\bar{\beta}}(t)dt + \big[(1-x)+xQ_{\bar{\beta}}(s)\big]\Big[\frac{\partial E[\boldsymbol{\mu}_s^2]}{\partial x}-\frac{\partial E[\boldsymbol{\mu}_s]}{\partial x}\Big].$$

Use $v^k(G_s)$ to make explicit that the interim utility function of the Sender corresponds to Motivation $k \in \{\mathrm{I}, \mathrm{II}\}$. Taking $s = s^*(x)$, readjust terms from the derivation of the (marginal) Sender's return from her evidence acquisition and disclosure, which was derived in the proof of Proposition 2, in Eq. (13) to obtain

$$MR^{\mathrm{k}}(s) = \int_s^{\bar{s}} \big[v^k(G_t)-v^k(G_s)\big]q_{\bar{\beta}}(t)dt + \big[(1-x)+xQ_{\bar{\beta}}(s)\big]\frac{dv^k(G_s)}{ds}\frac{ds}{dx}$$
$$= \int_s^{\bar{s}} \big[v^k(G_t)-v^k(G_s)\big]q_{\bar{\beta}}(t)dt + \big[(1-x)+xQ_{\bar{\beta}}(s)\big]\frac{\partial v^k(G_s)}{\partial x}$$

for each Motivation $k \in \{\mathrm{I}, \mathrm{II}\}$. Therefore, the difference $MR^{\mathrm{II}}(s) - MR^{\mathrm{I}}(s)$ can be expressed as

$$\int_s^{\bar{s}} \big[v^{II}(G_t)-v^{I}(G_t)+v^{I}(G_s)-v^{II}(G_s)\big]q_{\bar{\beta}}(t)dt + \big[(1-x)+xQ_{\bar{\beta}}(s)\big]\Big[\frac{\partial v^{II}(G_s)}{\partial x}-\frac{\partial v^{I}(G_s)}{\partial x}\Big].$$

Then, taking into account the preference specifications $v^I(G_s) = E[\boldsymbol{\mu}_s]$ and $v^{II}(G_s) = E[\boldsymbol{\mu}_s^2]$, we observe that the above derivation coincides exactly with the expression for $dW(x;s^*(x))/dx$ obtained previously. Therefore, $dW(x;s^*(x))/dx = MR^{\mathrm{II}}(s) - MR^{\mathrm{I}}(s) = MR^{\mathrm{II}}(x;s^*(x)) - MR^{\mathrm{I}}(x;s^*(x))$. ∎

Proof of Proposition 5. Consider Assumption 2. Consider two different distributions of priors $\boldsymbol{\beta}_i \sim F_i(\beta_i)$, $i = 1, 2$, such that $F_2(\beta_2)$ FOSD $F_1(\beta_1)$, with $\bar{\beta}_2 = \bar{\beta}_1 + \delta$, for an arbitrarily small $\delta \geq 0$. Consider any Motivation $k \in \{\mathrm{I}, \mathrm{II}\}$ for the Sender, described by her interim utility function $v_i(G_m^x)$ under the respective cdf of priors.

Take a given piece of evidence $s \in (\underline{s}, \hat{s})$. By using the expression for each $B_i(s)$ developed in the proof of Proposition 2 for each Motivation captured by $l \in \{1, 2\}$, it follows that

$$B_1(s) - B_2(s) =$$
$$\delta\Big[\pi_0(s)\int_0^1 F_2(\beta_s(\mu^{1/l}))d\mu - [\pi_0(s)-\pi_1(s)]\int_0^1 \big[F_2(\beta_s(\mu^{1/(l+1)})) - F_2(\beta_s(\mu^{1/l})\big]d\mu\Big]$$
$$+ (1-\bar{\beta}_1)\pi_0(s)\int_0^1 \big[F_1(\beta_s(\mu^{1/l})) - F_2(\beta_s(\mu^{1/l}))\big]d\mu$$
$$- q_{\bar{\beta}_1}(s)\Big[\int_0^1 \big[F_1(\beta_s(\mu^{1/(l+1)})) - F_2(\beta_s(\mu^{1/(l+1)}))\big]d\mu - \int_0^1 \big[F_1(\beta_s(\mu^{1/l})) - F_2(\beta_s(\mu^{1/l}))\big]d\mu\Big].$$

Consider $\delta \approx 0$ so that the first term in the expression above vanishes. Then, by dividing the resulting expression over $q_{\bar{\beta}_1}(s)$, we obtain that $B_1(s) > B_2(s)$ if and only if

$$\frac{\int_0^1 \big[F_1(\beta_s(\mu^{1/(l+1)})) - F_2(\beta_s(\mu^{1/(l+1)}))\big]d\mu}{\int_0^1 \big[F_1(\beta_s(\mu^{1/l})) - F_2(\beta_s(\mu^{1/l}))\big]d\mu} < 2 - \mu_s(\bar{\beta}_1),$$

as stated. ∎

Proof of Proposition 6. Consider Assumption 2 and take an interior equilibrium $(x^*, s^*(x^*))$. In the main text, we have shown that the sign of dx^*/dr coincides with the sign of $\partial MR(x^*, s^*(x^*))/\partial r$. Therefore, our goal is to study the sign of the marginal change $\partial MR(x^*, s^*(x^*))/\partial r$. To proceed, we find convenient to introduce two expressions that capture the features of the shape of the Sender's interim utility with respect to, respectively, priors and the evidence acquisition effort. First, consider the expectation $z(G_{s;r}) \equiv E\big[S_r(\beta)[\partial^2\mu_s^l(\beta)/\partial\beta^2]\big]$ where $l = 1$ for Motivation I and $l = 2$ for Motivation II. Specifically, $z(G_{s;r})$ is an adjusted measure of the degree of convexity of the Sender's interim utility at equilibrium with respect to β. Such a measure is weighted by the size of mean-preserving spread in Eq. (7). Second, using such a measure, we then define the normalized marginal interim utility $\tilde{\chi}(s;r) \equiv -[dz(G_{s;r})/ds]/[\lambda'(s)/\lambda(s)]$. Specifically, $\tilde{\chi}(s;r)$ measures the degree of concavity/convexity of the marginal change of the Sender's interim utility with respect to the effort x. We have $\tilde{\chi}(s;r) = -E[S_r(\beta)[\partial^2\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)/\partial\beta^2]]$ for Motivation I and $\tilde{\chi}(s;r) = -2E[S_r(\beta)[\partial^2\boldsymbol{\mu}_s^2(1-\boldsymbol{\mu}_s)/\partial\beta^2]]$ for Motivation II.

We first note that

$$\frac{\partial MR(x^*, s^*(x^*))}{\partial r} = \left[(1-x^*) + x^* Q_{\bar{\beta}}(s^*(x^*))\right] \int_0^1 \frac{\partial \mu_{s^*(x^*)}}{\partial x} \frac{\partial^2 F_r(\beta)}{\partial \beta \partial r} d\beta + \int_{s^*(x^*)}^{\bar{s}} q_{\bar{\beta}}(s) \int_0^1 [\mu_s - \mu_{s^*(x^*)}] \frac{\partial^2 F_r(\beta)}{\partial \beta \partial r} d\beta ds. \tag{16}$$

We then integrate the first term in Eq. (16) above by parts. To do so, we use $u = \partial \mu_{s^*(x^*)}/\partial x$ and $dv = [\partial^2 F_r(\beta)/\partial \beta \partial r] d\beta$ together with $\partial F_r(0)/\partial r = \partial F_r(1)/\partial r = 0$. To see why such a latter property holds, note that for two cdfs we have $F_{r_1}(\beta) - F_{r_0}(\beta) = 0$ for $\beta \in \{0, 1\}$, which, for an infinitesimal change in r, translates into $\partial F_r(0)/\partial r = \partial F_r(1)/\partial r = 0$. Thus, we obtain

$$\int_0^1 \frac{\partial \mu_{s^*(x^*)}}{\partial x} \frac{\partial^2 F_r(\beta)}{\partial \beta \partial r} d\beta = -\int_0^1 \frac{\partial^2 \mu_{s^*(x^*)}}{\partial x \partial \beta} \frac{\partial F_r(\beta)}{\partial r} d\beta.$$

We integrate again the expression above by parts using now $u = \partial^2 \mu_{s^*(x^*)}/\partial x \partial \beta$ and $dv = [\partial F_r(\beta)/\partial r] d\beta$, together with $S_r(0) = S_r(1) = 0$. Here, $S_r(0) = 0$ follows directly by plugging $y = 0$ in Eq. (7), whereas $S_r(1) = 0$ is implied by the mean-preserving property. Formally, for two cdfs $F_{r_0}(\beta)$ and $F_{r_1}(\beta)$, we have (after integrating by parts) that

$$\int_0^1 \beta \{dF_{r_1}(\beta) - dF_{r_0}(\beta)\} d\beta = -\int_0^1 [F_{r_1}(\beta) - F_{r_0}(\beta)] d\beta + \beta [F_{r_1}(\beta) - F_{r_0}(\beta)] \Big|_{\beta=0}^{\beta=1} = 0.$$

We have thus shown that the second term of the above derivation equals zero. The first term equals zero as well since it coincides with $-S_r(1)$ in Eq. (7). For an infinitesimal change in r, the mean-preserving property translates then into

$$\int_0^1 \beta \frac{\partial F_r(\beta)}{\partial r} d\beta = -\int_0^1 \frac{\partial F_r(\beta)}{\partial r} d\beta + \beta \frac{\partial F_r(\beta)}{\partial r} \Big|_{\beta=0}^{\beta=1} = 0.$$

We therefore obtain

$$-\int_0^1 \frac{\partial^2 \mu_{s^*(x^*)}}{\partial x \partial \beta} \frac{\partial F_r(\beta)}{\partial r} d\beta = \int_0^1 \frac{\partial^3 \mu_{s^*(x^*)}}{\partial x \partial \beta^2} S_r(\beta) d\beta.$$

Proceeding in a totally analogous way, the second term in Eq. (16) can be expressed as

$$\int_{s^*(x^*)}^{\bar{s}} q_{\bar{\beta}}(s) \int_0^1 \frac{\partial^2}{\partial \beta^2} \left[\mu_s - \mu_{s^*(x^*)}\right] S_r(\beta) d\beta.$$

Now, combine the two expressions above to write $\partial MR(x^*, s^*(x^*))/\partial r$ as

$$\int_{s^*(x^*)}^{\bar{s}} q_{\bar{\beta}}(s)[z(G_{s;r}) - z(G_{s^*(x^*);r})]ds + \left[(1-x^*) + x^* Q_{\bar{\beta}}(s^*(x^*))\right] \frac{dz(G_{s^*(x^*);r})}{ds}\frac{ds^*}{dx}. \tag{17}$$

First, for Motivation I, we have $z(G_{s^*(x^*);r}) = E\left[\frac{\partial^2 \boldsymbol{\mu}_{s^*(x^*)}}{\partial \beta^2} S_r(\beta)\right] > 0$ since posteriors are convex in β at $s^*(x^*) < \hat{s}$ and $S_r(\beta) > 0$ for $\beta \in \mathcal{B}$. Let the equilibrium threshold be $s \equiv s^*(x^*)$, then by replicating the arguments used in the proof of Proposition 2, the first term in Eq. (17) equals $[z(G_{\bar{s};r}) - z(G_{s;r})]Q_{\bar{\beta}}(s)$. Note that $z(G_{\bar{s};r}) = 0$ since by Assumption 2, we are considering $\pi_0(\bar{s}) = 0$ and $\pi_1(\bar{s}) > 0$ when $\bar{s} < +\infty$. The second term in Eq. (17) becomes

$$-\frac{dz(G_{s;r})}{ds}\frac{\lambda(s)}{\lambda'(s)} q_{\bar{\beta}}(s)\rho_{\xi}(s) = \tilde{\chi}(s;r) q_{\bar{\beta}}(s)\rho_{\xi}(s),$$

where $\rho_{\xi}(s) = IH_1(s) - IH_0(s)$ and $\tilde{\chi}(s,r) = -E\left[S_r(\beta)\frac{\partial^2}{\partial \beta^2}\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)\right]$. We thus have

$$-z(G_{s;r})[1 + (1-\bar{\beta})\pi_0\rho_{\xi}(s) - q_{\bar{\beta}}(s) IH_1(s)] + \rho_{\xi}(s) q_{\bar{\beta}}(s)\tilde{\chi}(s,r).$$

Simple algebra (similarly as done in the proof of Proposition 2) shows that the term that accompanies $-z(G_{s;r})$ amounts to $Q_{\bar{\beta}}(s)$. Therefore,

$$\frac{\partial MR(s)}{\partial r} = -z(G_{s;r})Q_{\bar{\beta}}(s) + q_{\bar{\beta}}(s)\tilde{\chi}(s;r)\rho_{\xi}(s). \tag{18}$$

The first term in Eq. (18) is negative and the sign of the second term of Eq. (18) is determined by the sign of $\tilde{\chi}(s,r) = -E\left[S_r(\beta)\frac{\partial^2}{\partial \beta^2}\boldsymbol{\mu}_s(1-\boldsymbol{\mu}_s)\right]$, which we study in what follows. Note that

$$\frac{\partial^2}{\partial \beta^2}\mu_s(1-\mu_s) = \pi_1(s)\pi_0(s)\frac{\partial^2}{\partial \beta^2}\frac{\beta(1-\beta)}{q_{\bar{\beta}}(s)^2}$$

and

$$\frac{\partial^2}{\partial \beta^2}\frac{\beta(1-\beta)}{q_{\bar{\beta}}(s)^2} = \frac{-2[\beta[\pi_0(s)^2 - \pi_1(s)^2] + \pi_0(s)[2\pi_1(s) - \pi_0(s)]}{q_{\bar{\beta}}(s)^4}. \tag{19}$$

Thus, there exists a threshold

$$\beta^{\mathrm{I}}(s;r) = \frac{\pi_0(s)[\pi_0(s) - 2\pi_1(s)]}{[\pi_0(s)^2 - \pi_1(s)^2]} < 1$$

such that if $\beta < \beta^{\mathrm{I}}(s;r)$, then $\partial^2\mu_s(1-\mu_s)/\partial\beta^2 > 0$. Given that $S_r(\beta) > 0$ for $\beta \in \mathcal{B}$, we can conclude that $\tilde{\chi}(s;r) < 0$. Then, for $F(\beta^{\mathrm{I}}(s;r)) \approx 1$, the monotonicity property in Theorem 16.1 in Billingsley (1995) guarantees that $\partial MR(s)/\partial r < 0$. If $\tilde{\chi}(s;r) > 0$ then $\partial MR(s)/\partial r > 0$ if and only if

$$\frac{\tilde{\chi}(s;r)}{z(s;r)} > \frac{Q_{\bar{\beta}}(s)}{q_{\bar{\beta}}(s)\rho_{\xi}(s)}.$$

We use the equilibrium expression for $z(s;r)$ and $\tilde{\chi}(s;r)$, presented in the main text, to obtain the formulation proposed in the proposition.

For Motivation II, the analysis of Eq. (18) is totally analogous. To simplify subsequent exposition, we set the ratio $C \equiv q_{\bar{\beta}}(s)(1-\lambda(s))/\pi_0(s)\lambda(s)$. It can be verified that $C > 0$ since $\lambda(s) < 1$ at the equilibrium threshold $s \equiv s^*(x^*)$. It follows that $\partial^2\mu_s/\partial\beta^2 = 2[\partial\mu_s/\partial\beta]^2 C > 0$ and, thus, $z(G_{s;r}) = E\left[\frac{\partial^2 \boldsymbol{\mu}_s^2}{\partial\beta^2} S_r(\beta)\right] > 0$. In addition,

$$\frac{\partial^2\mu_s^2}{\partial\beta^2} = 2\left[\left[\frac{\partial\mu_s}{\partial\beta}\right]^2 + \mu_s\frac{\partial^2\mu_s}{\partial\beta^2}\right] = 2\left[\frac{\partial\mu_s}{\partial\beta}\right]^2[1+2\mu_s C] > 0.$$

The sign of $\tilde{\chi}(s;r) = -2E\left[S_r(\beta)\frac{\partial^2}{\partial\beta^2}\boldsymbol{\mu}_s^2(1-\boldsymbol{\mu}_s)\right]$ is determined by the sign of

$$\frac{\partial^2}{\partial\beta^2}\mu_s^2(1-\mu_s) = \frac{\partial^2}{\partial\beta^2}\left[\mu_s\frac{\beta(1-\beta)}{q_{\bar{\beta}}(s)^2}\right].$$

Note that

$$\frac{\partial^2}{\partial\beta^2}\left[\mu_s\frac{\beta(1-\beta)}{q_{\bar{\beta}}(s)^2}\right] = \left[\frac{\partial^2\mu_s}{\partial\beta^2}\right]\frac{\beta(1-\beta)}{q_{\bar{\beta}}(s)^2} + 2\frac{\partial\mu_s}{\partial\beta}\frac{\partial}{\partial\beta}\left[\frac{\beta(1-\beta)}{q_{\bar{\beta}}(s)^2}\right] + \mu_s\frac{\partial^2}{\partial\beta^2}\left[\frac{\beta(1-\beta)}{q_{\bar{\beta}}(s)^2}\right].$$

Use the previous derivation $\partial^2\mu_s/\partial\beta^2 = 2[\partial\mu_s/\partial\beta]^2 C > 0$ together with the equality

$$\frac{\partial}{\partial\beta}\frac{\beta(1-\beta)}{q_{\bar{\beta}}(s)^2} = \frac{-\beta[\pi_1(s)+\pi_0(s)]+\pi_0(s)}{q_{\bar{\beta}}(s)^3}$$

to rewrite the above expression for $\frac{\partial^2}{\partial\beta^2}\left[\mu_s\frac{\beta(1-\beta)}{q_{\bar{\beta}}(s)^2}\right]$ as

$$\frac{\partial^2}{\partial\beta^2}\left[\mu_s\frac{\beta(1-\beta)}{q_{\bar{\beta}}(s)^2}\right] = \left[2\frac{\partial\mu_s}{\partial\beta}\right]\left[\frac{\beta(1-\beta)}{\pi_0(s)q_{\bar{\beta}}(s)}\frac{\partial\mu_s}{\partial\beta}\left[\frac{1}{\lambda(s)}-1\right] + \frac{\partial}{\partial\beta}\left[\frac{\beta(1-\beta)}{q_{\bar{\beta}}(s)^2}\right]\right] + \mu_s\frac{\partial^2}{\partial\beta^2}\left[\frac{\beta(1-\beta)}{q_{\bar{\beta}}(s)^2}\right]$$

$$= \left[2\frac{\partial \mu_s}{\partial \beta}\right]\left[\frac{\beta(1-\beta)}{q_{\bar{\beta}}(s)}\frac{\partial \mu_s}{\partial \beta}\left[\frac{\pi_0(s)-\pi_1(s)}{\pi_1(s)\pi_0(s)}\right] + \frac{-\beta(\pi_1(s)+\pi_0(s))+\pi_0(s)}{q_{\bar{\beta}}(s)^3}\right]\right] + \mu_s\frac{\partial^2}{\partial \beta^2}\left[\frac{\beta(1-\beta)}{q_{\bar{\beta}}(s)^2}\right]$$

$$= \left[2\frac{\partial \mu_s}{\partial \beta}\right]\left[\left[\frac{\beta(1-\beta)[\pi_0(s)-\pi_1(s)] - \beta(\pi_1(s)+\pi_0(s))+\pi_0(s)}{q_{\bar{\beta}}(s)^3}\right] + \mu_s\frac{\partial^2}{\partial \beta^2}\left[\frac{\beta(1-\beta)}{q_{\bar{\beta}}(s)^2}\right].$$

Finally, use Eq. (19) together with $\frac{\partial \mu_s}{\partial \beta} = \frac{\pi_1(s)\pi_0(s)}{q_{\bar{\beta}}(s)^2}$ and μ_s itself, to rewrite the expression above as $[2\pi_1(s)\pi_0(s)/q_{\bar{\beta}}(s)^5]P(\beta)$, where

$$P(\beta) \equiv$$
$$\pi_0(s) + \beta\pi_1(s)[-2+\pi_0(s)[\pi_0(s)-2\pi_1(s)]] - \beta^2[\pi_0(s)-\pi_1(s)][\pi_1(s)[\pi_0(s)+\pi_1(s)]+1].$$

The function $P(\beta)$ is a concave and continuous polynomial of degree two that takes values $P(0) = \pi_0(s) > 0$ and $P(1) = \pi_1[\pi_1(s)[\pi_1(s) - 2\pi_0(s)] - 1] < 0$. Therefore, by the intermediate value theorem, there exists a threshold $\beta^{\mathrm{II}}(s;r) \in (0,1)$ such that $[2\pi_1(s)\pi_0(s)/q_{\bar{\beta}}(s)^5]P(\beta)$ is positive if and only if $\beta < \beta^{\mathrm{II}}(s;r)$. Equivalently, for such an expression to be positive we just need to require that $F(\beta^{\mathrm{II}}(s;r)) \approx 1$. In this case, $\tilde{\chi}(s;r) < 0$ and $\partial MR(s)/\partial r < 0$. If $\tilde{\chi}(s;r) > 0$, then $\partial MR(s)/\partial r > 0$ if and only if

$$\frac{\tilde{\chi}(s;r)}{z(s;r)} > \frac{Q_{\bar{\beta}}(s)}{q_{\bar{\beta}}(s)\rho_{\xi}(s)}.$$

We conclude the required arguments by making use of the expressions for $z(s;r)$ and $\tilde{\chi}(s;r)$ at the equilibrium threshold $s = s^*(x^*)$. ∎